1971

Study of highway drainage inlets

George M. Lee
Lehigh University

STUDY OF HIGHWAY DRAINAGE INLETS

by

George M. Lee

ABSTRACT

A theoretical study supplemented with experimental data for development of improved drainage inlets was made.

Two different approaches were applied to the spatially varied flow with increasing discharge using the energy and momentum principles. Dynamic differential equations were reached. Two different methods to the problem of flow into drainage inlets were applied, namely, the specific energy method and a rather simplified method. The final dynamic equation was solved numerically by using a digital computer. An experimental study on the resistance coefficient for exterior plywood was also made. Einstein's method was used for the exclusion of side glass effect on the rectangular test channel.

The results show that water flowing in highway gutters is accurately described by using the theory of spatially varied flow. A computer program solving the differential equation involved for the case of rectangular, trapezoidal, or rectangular shapes of gutters is also given.

CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

2 SEPTEMBER 1970
(Date)

Arthur W Brune PE
Professor Arthur W. Brune
Professor in Charge

David A VanHorn
Professor David A. VanHorn
Chairman, Department of
Civil Engineering

STUDY OF HIGHWAY DRAINAGE INLETS

by

George M. Lee

A Thesis
Presented to the Graduate Committee
of Lehigh University
in Candidacy for the Degree of
Master of Science

in

Civil Engineering

Lehigh University

1970

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

This thesis presents partial results of the project entitled "Development of Improved Drainage Inlets" conducted at the Fritz Engineering Laboratory of Lehigh University, Bethlehem, Pennsylvania. Dr. Arthur W. Brune is director of this research project. Dr. Lynn S. Beedle is Director of Fritz Engineering Laboratory and Dr. David A. VanHorn is the Chairman of the Department of Civil Engineering.

The writer wishes to thank Dr. Arthur W. Brune for his interest and guidance throughout the investigation and for reviewing this paper. Special thanks are due to Dr. Osman A. ElGhamry for his valuable suggestions during the writing of this manuscript.

The writer is indebted to Mrs. Jane L. Lenner who typed the manuscript with great care and to Mr. John M. Gera, Jr. who prepared the drawings.

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

A theoretical study supplemented with experimental data for development of improved drainage inlets was made.

Two different approaches were applied to the spatially varied flow with increasing discharge using the energy and momentum principles. Dynamic differential equations were reached. Two different methods to the problem of flow into drainage inlets were applied, namely, the specific energy method and a rather simplified method. The final dynamic equation was solved numerically by using a digital computer. An experimental study on the resistance coefficient for exterior plywood was also made. Einstein's method was used for the exclusion of side glass effect on the rectangular test channel.

The results show that water flowing in highway gutters is accurately described by using the theory of spatially varied flow. A computer program solving the differential equation involved for the case of rectangular, trapezoidal, or rectangular shapes of gutters is also given.

1. INTRODUCTION

Runoff from rainfall must be removed from the highway. This is done by placing drainage inlets at intervals along the roadside, as shown in Fig. 1. The most efficient position of the inlets is determined particularly by the drainage capacity of the inlets and by the approach conditions. An efficient design should avoid permitting water to overflow the inlet, because sufficient repetitions of overflowage could lead to flooding of the highway. This would cause not only traffic to stop but damage to the highway pavement and subgrade as well. The numerous variables involved in a hydraulic study of drainage inlets make it clear that a theoretical solution to this study is impossible. However, a theoretical study supplemented by an experimental study using either prototype or model inlets probably would give a proper solution to this problem.

Hydrological studies involve determination of the amount of runoff to be removed by the inlets as a result of the rainfall in the vicinity of the highway. Obviously, such a study is extremely important in the design of drainage inlets. However, within this report only the hydraulics of highway drainage inlets will be considered.

2. OBJECTIVE AND DESCRIPTION OF THE PROBLEM

The main objective of this study is to develop an ideal design for inlets, which has high efficiency. Both theoretical and experimental approaches are needed. The procedure followed is to determine the theoretical dimension of the inlet required for the complete withdrawal of the flow in the upstream gutter. In order to achieve that goal the following problems must be solved:

1. The determination of the critical section of the gutter;

2. The development of appropirate formulas or equations to describe the flow in the gutter;

3. The determination of a method to describe the flow at the inlet, and

4. The determination of adjusting coefficients by experimental investigations.

The results of Item 1 can be used as the boundary conditions for the second item. Using the known velocity and the known water depth at the section, where critical conditions take place, as initial values will enable one to compute the velocity and water profile along the gutter using the equations obtained in Item 2. The third problem can be analyzed either by the simplified method which assumes a freely falling body of water in the gutter in front of the inlet or by the specific energy method which assumes that the specific energy is constant along the gutter for spatially varied flow. In Item 3 certain parameters and coefficients, which change with the characteristics of the inlet and the flow conditions in the gutter, cannot be obtained theoretically. This requires the experimental determination of these parameters, which can be done by using either prototype or model studies. The details of the method of model design will not be included in this study.

Investigations made to date (12,19,29) have yielded information for some aspects of the inlet design. Meaningful points are the following items:

1. Maximal efficiency and significant economy can be attained if 5% to 10% of the flow is allowed to pass over the inlet.

2. The capacity of the inlet is proportional to the overall dimensions thereof.

3. Inlets with the bars parallel to the direction of flow in the gutter have higher efficiency and self-cleaning ability than inlets wherein the bars are not parallel to the direction of the flow.

4. Curb or side inlets are inefficient in comparison to drop inlets, unless clogging of an inlet with debris is a serious problem. Combination inlets give the highest efficiencies if appreciable clogging occurs.

5. The efficiency of an inlet decreases as its slope increases.

There are many other researches which are indirectly related to highway drainage systems, such as those dealing with economics, with overland flow, and with infiltration. If runoff from the backslope is considered in determining the capacity of inlets, studies of surface flow over permeable material, such as soil, are significant. Such investigations, however, are not included in this study.

3. ANALYTIC APPROACH

The open channel flow in a highway gutter can be classified into three categories: (1) supercritical flow, (2) a mixture of super-critical and subcritical flow, and (3) subcritical flow. Each category of flow has its own special hydraulic characteristics. Therefore, the study of this problem should be divided accordingly. From the point of view of the highway engineer inlets can be designed with little concern about the kind of flow involved, although it has a certain effect on the performance of the drainage inlets, which effect will be discussed in Section 3.4.

From previous investigations (16) in which longitudinal slopes were equal to or greater than 0.01 were examined, the flow was noted to be usually supercritical. Considering the slopes that most possibly will encounter in the field, supercritical flow probably will occur. More detailed remarks about this point are discussed in Section 3.4.2; additionally, an example of highway gutter flow is given in Section 3.2.4.

3.1 The Friction Formula

At the end of the 19th Century and at the beginning of the 20th Century engineers proposed many different formulas for relating the mean velocity of water to both the energy gradient and the hy-draulic radius. Only a few of them are still in use in 1970. One formula which was proposed by an Irish engineer named Robert Manning in 1880 (4) is known as the Manning formula. Due to its simplicity of form and a lot of data readily available, it has become the most widely used formula for uniform, open-channel flow calculations. This formula was later modified with English units to:

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{1}$$

in which V is the mean velocity, fps,

S is the slope of the energy line,

R is the hydraulic radius, ft., and

n is the resistance coefficient, usually termed Manning's n.

Despite the simplicity of Eq. (1), it has the distinct drawback in that n is not dimensionless but has the dimensions $TL^{-1/3}$. However, in using this formula trial and error methods are not involved. But the correct determination of n requires considerable judgement and experience. Furthermore, that although n was thought to vary with the channel roughness only, yet, it was observed from laboratory and field data that it additionally depends in some unknown manner upon the size and shape of the channel, the rate of discharge, and the depth of water in the channel. A better understanding of the factors that affect the resistance coefficient would give some help for the proper selection of n. Values of Manning's n, determined by many tests on actual canals and channels, are given in most books on hydraulics (3,6,16,20).

The Manning formula is restricted primarily to steady uniform flow. It would be a very crude approximation to assume that flow in a highway gutter fits the above restrictions. Because of the seasonal growth of grass and weeds in a channel or along highway embankments together with the existence of both suspended material and bed-load in the gutter flow, the roughness coefficient should change with the seasons. Generally, conditions which tend both to induce turbulence and to cause friction will increase n and those conditions which tend to reduce turbulence will decrease n. However, the change in the roughness pattern due to all of these factors can be encompassed by choosing n properly.

3.2 Theory of Spatially Varied Flow

In an open-channel where the water enters or leaves the channel such flow is known as spatially varied flow. This type of flow occurs very commonly in roadside gutters. Water flowing in a channel must conform to the two laws of motion: The law of conservation of linear momentum and the law of conservation of energy. For open-channel hydraulics the first law is commonly expressed as "force is equal to the rate of change of momentum with respect to time". The law of conservation of energy is commonly expressed in hydraulics in the form of the Bernoulli equation, which states that the sum of the elevation head, the pressure head, and the velocity head is equal to the total head at any other point in the flow, less the internal losses. Those losses come from the transformation of kinetic or static energy into heat, which energy is not actually destroyed.

As far as this study is concerned, neither of these laws are inherently superior to the other. Proper application of either law always yields similar results. In some hydraulic problems one or the other might be more readily applicable. Before deriving the dynamic equation of spatially varied flow with increasing discharge, the following assumptions are made:

1. The velocity distribution across the gutter section is uniform;

2. The pressure within the flow is hydrostatic; and

3. The effect of air entrainment is neglegible.

3.2.1 Momentum Approach

In highway gutter flow a constant increment of rainfall inflow per unit length, q, is assumed to occur; further, the momentum of the inflow is neglected for simplicity. For a changing velocity within a control volume the momentum principle states that at any instant the rate of change of linear momentum is equal to the resultant force acting on the volume. A control volume is a volume fixed in the space relative to a coordinate system. With reference to Fig. 2a consider two vertical cross sections at a distance, δx, apart. If the velocity distribution in the gutter section is not constant and is non-uniform, a momentum correction coefficient, α, can be introduced. The momentum at section 1 per unit time is

$$M_1 = \frac{\alpha\gamma}{g} QV \qquad (2)$$

in which γ is the unit weight of water, pcf,

g is the acceleration of gravity, fpsps, and

Q is the discharge, cfs.

Provided the α is constant in the gutter, similarly, the moment at section 2 per unit time is

$$M_2 = \frac{\alpha\gamma}{g} (Q + q \delta x) (V + \delta V) \tag{3}$$

where q δx is the added inflow between sections 1 and 2. The change of momentum in the distance δx is

$$\delta M = M_2 - M_1 = \frac{\alpha\gamma}{g} [Q \delta V + q \delta x (V + \delta V)] \tag{4}$$

As the size of the control volume decreases, incremental notation is replaced by differential notation which leads to

$$dM = \frac{\alpha w}{g} [Q dV + q dx (V + dV)] \tag{5}$$

The differential product, dxdV, is an infinitesimal of 2nd order and therefore can be neglected, the result is

$$dM = \frac{\alpha w}{g} [Q dV + q V dx] \tag{6}$$

The net force in the direction of flow is the difference between the hydrostatic force and the component of the weight of water in the direction of flow less the frictional force. The frictional force is equivalent to the shear stress, τ_o, multiplied by the wetted **area,** P dx, in the control volume, or

$$F_f = \tau_o \, P \, dx \tag{7}$$

in which P is the wetted perimeter. In open-channel flow τ_o can be replaced by $\gamma \, S_f \, R$ (3), thus

$$F_f = \gamma \, S_f \, R \, P \, dx \text{ , or}$$

$$F_f = \gamma \, S_f \, A \, dx \tag{8}$$

in which S_f is the friction slope. By using the Manning formula (11) S_f can be expressed as

$$V = \frac{1.49}{n} R^{2/3} \, S_f^{1/2} \text{ , or}$$

$$S_f = \frac{Q^2 \, n^2}{2.21 \, A^2 \, R^{4/3}} \tag{9}$$

The weight of water in the control volume, W, acting in the direction of flow is

$$W \sin\theta = \gamma \, (A + 1/2 \, dA) \, dx \sin\theta \text{ , or} \tag{10}$$

$$W \sin\theta = \gamma \, S_o \, A \, dx \tag{11}$$

In Eq. (10) the $\sin\theta$ is approximated by S_o, which is the bed slope of the gutter. The product of differential dAdx is dropped for simplicity. Referring to Fig. 2c, the change in hydrostatic force from section 1 to section 2 is

$$F_1 - F_2 = -\gamma A\, dy \tag{12}$$

by neglecting terms containing high orders of dy. It is noteworthy that the bed slope has no direct effect upon the net hydrostatic force. Equating the momentum change of the control volume, Eq. (6), to all the external forces acting on the water, using left hand side of Eqs. (8), (11), and (12), leads to:

$$\frac{d\gamma}{g}\,[Q\, dV + q\, V\, dx] = -F_f + W \sin\theta + F_1 - F_2 \tag{13}$$

Substituting right hand side of Eqs. (8), (11), and (12) into the above equation and dividing by γA yields:

$$dy = -\frac{\alpha V}{g}\,(dV + \frac{q}{A}\, dx) + (S_o - S_f)\, dx \tag{14}$$

Noting that $V = \frac{Q}{A}$ and $dV = -V + \frac{Q + dQ}{A + dA}$, the above equation can be written as:

$$dy = -\frac{\alpha V}{g}\;\frac{2A\, q\, dx - QdA + q\, dA\, dx}{A\,(A + dA)} + (S_o - S_f)\, dx \tag{15}$$

In order to simplify the above equation, q dA dx, in the numerator and, dA, in the denominator is neglected; then dividing by dx

$$\frac{dy}{dx} = -\frac{\alpha V}{g}\left(\frac{2q}{A} - \frac{Q}{A^2}\frac{dA}{dx}\right) + (S_o - S_f) \qquad (16)$$

Referring to Fig. 2c, the change of water area with respect to x can be obtained as:

$$A = \frac{1}{2} y\, T \text{ , and}$$

$$\frac{dA}{dx} = \frac{T}{2}\frac{dy}{dx} + \frac{y}{2}\frac{dT}{dx} \qquad (17)$$

Equation (17) can be further simplified to

$$\frac{dA}{dx} = T\frac{dy}{dx} \text{ , or} \qquad (18)$$

$$\frac{dA}{dx} = \frac{A}{y_m}\frac{dy}{dx} \qquad (19)$$

where y_m is the hydraulic depth in the gutter. Substituting Eq. (19) into Eq. (16) and simplifying, the equation becomes

$$\frac{dy}{dx} = \frac{S_o - S_f - 2\alpha Qq/gA^2}{1 - \alpha Q^2/gA^2\; y_m} \qquad (20)$$

The above equation is the usual differential equation for the change in depth upon going downstream in a channel and in which the discharge increases with distance.

3.2.2 Energy Approach

Due to the unpredictable energy loss of turbulent mixing of the lateral inflow and the water flowing in the gutter, the momentum approach is normally used for the derivation of the differential equation of spatially varied flow. However, it is much more convenient to deal with the more general case by means of the energy approach.

Referring to Fig. 2c, the lateral inflow is qdx in a length of dx. The lateral inflow entering the gutter with a zero longitudinal velocity in the small stretch dx must be accelerated to the velocity of V + dV upon leaving the control volume. The energy required to accelerate the flow from zero to V + dV is

$$\frac{1}{2}\,\beta M\,(V + dV)^2 \cong \beta M\,V^2 \tag{21}$$

where M is the mass of flowing water within the control volume,

and

β is the energy correction coefficient.

The energy required per pound of water is

$$\frac{\frac{1}{2g}\,(\gamma\, q\; dx\; dt)\;\beta\, V^2}{\gamma A\; dx} = \frac{q}{2g}\,\frac{\beta\, V^2}{A}\,dt \tag{22}$$

In order to accelerate the inflow from zero to V + dV in time dt and over a distance dx, the following equation of motion should be satisfied:

$$\frac{1}{2}(V + dV)\,dt = dx \quad \text{, or approximately}$$

$$\frac{1}{2} V\,dt = dx \tag{23}$$

Introducing dt obtained from Eq. (23) into Eq. (22) yields the energy required per pound of water as:

$$\frac{\beta V}{gA} q\,dx \quad \text{, or}$$

$$\frac{\beta Q}{gA^2} q\,dx \tag{24}$$

This may be expressed for an infinitesimal longitudinal dixtance dx as:

$$\frac{\beta Q}{gA^2} q \tag{25}$$

The energy equation for the control volume can be written as:

$$E = d + \beta \frac{Q^2}{2g\,A^2} + \text{(energy required to accelerate inflow)} \tag{26}$$

Differentiating this equation with respect to x and introducing Eq. (25) for the last term, yields:

$$\frac{dE}{dx} = \frac{dd}{dx} + \frac{\beta}{2g}\left(\frac{2Q}{A^2} q - \frac{2Q^2}{A^3}\frac{dA}{dx}\right) + \frac{\beta Q}{gA^2} q \tag{27}$$

The friction slope can be expressed by the Manning formula as has been done in the momentum approach, and its direction is always opposite to the direction of flow, thus

$$S_f = \frac{Q^2 n^2}{2.21 A^2 R^{4/3}}, \text{ or} \tag{28}$$

$$S_f = - \frac{dE}{dx} \tag{29}$$

Inserting Eq. (29) into Eq. (27) and rearranging terms, the result is the general equation for spatially varied flow with increasing discharge.

$$\frac{dd}{dx} = - \frac{2\beta Q}{gA^2} q + \frac{\beta Q^2}{gA^3} \frac{dA}{dx} - S_f \tag{30}$$

This equation can be used for any shape of channel.

For the particular case of gutter flow with an isosceles cross section Eq. (30) can be further simplified. Referring to Fig. 3c, the elevation of the water surface with respect to the datum is given by

$$d = z + y \tag{31}$$

where z is the elevation of the gutter bottom above the datum. The changing of d with respect to x can be obtained by differentiating Eq. (31) with respect to x,

$$\frac{dd}{dx} = \frac{dz}{dx} + \frac{dy}{dx} \text{ , or} \tag{32}$$

$$\frac{dd}{dx} = - S_o + \frac{dy}{dx} \tag{33}$$

Using Eqs. (19) and (33), Eq. (30) can be written as:

$$\frac{dy}{dx} = - \frac{2\beta Q}{gA^2} q + \frac{\beta Q^2}{gA^2} \frac{1}{y_m} \frac{dy}{dx} + S_o - S_f \tag{34}$$

After rearranging the terms the final differential equation becomes

$$\frac{dy}{dx} = \frac{S_o - S_f - \dfrac{2\beta Q q}{gA^2}}{1 - \dfrac{\beta Q^2}{gA^2 \; y_m}} \tag{35}$$

Equations (20) and (35) seem to be almost identical, but they are literally not. Although both approaches are derived from Newton's second law of motion, due to the basic difference in physical concepts they are inherently different. The momentum approach is a vector relationship which considers only the momentum force changing in the direction of flow, whereas the energy approach is a scalar relationship that equates the algebraic sum of all the energy added or dissipated within the control volume. Both equations are almost identical because they have been simplified. For instance, the momentum-flux correction factor, α, appears only in the momentum equation, whereas the kinetic-energy flux correction factor, β, appears only in the energy equation. They are not necessarily equal to each other; this means that using the average flow velocity will not have the same effect on both momentum and energy. Furthermore, the friction slope, S_f, can also be described by either the Chezy or the Darcy-Weisbach resistance formula. For the Chezy formula, $S_f = \frac{Q^2}{C^2 R A^2}$, and for the Darcy-Weisbach formula, $S_f = \frac{f Q^2}{8g A^2 R}$, where C is a factor of flow resistance, called Chezy's coefficient, and f is the Darcy-Weisbach friction factor. For a steady, spatially varied flow, in the momentum approach, the friction slope should be used. While in the energy approach, the energy gradient should be used which is identical to the friction slope in the momentum approach. It is because of the basic difference in the physical concepts of the two approaches, for the same slope there are two different names for it. For a given problem the two slopes should have the same numerical values provided the same friction formula is being used. In the computation of dy/dx the final result is directly dependent upon whether the Manning, Chezy, or Darcy-Weisbach formulas are being used as well as what approach is under consideration. Although a certain difference lies between different combinations, the derivation can always be considered as a minor one. Both momentum and energy approaches discussed in this section can only be considered as simplified methods of attacking this problem. For a detailed discussion of the pressure correction factor and of the potential-energy flux correction factor reference should be made to Yen and Wenzel (32).

3.2.3 Determination of the Control Section

The information given in the previous section permits computing the water surface profile in a channel with spatially varied flow that has an increasing discharge. Before beginning the calculation it is necessary to select the control section from which the flow-profile computation can start. Usually, the section where critical condition takes place is determined. The water depth and average velocity at that section are known and equal to critical depth and critical velocity. These known values will serve as an initial condition for the water profile computation.

The determination of the control section presents some difficulties. A method for locating the critical depth was first proposed by J. Hinds (4), using an equivalent "critical depth channel". An alternative method requiring fewer calculations was presented by F. F. Escoffier (9), and further modified by K. V. H. Smith (26) for spatially varied flow. In comparison with the Hinds method, the Smith method contains fewer computations and is applicable to gutters of either uniform or non-uniform cross section.

The final equation obtained from Smith's approach is

$$\frac{1}{Q}\frac{dQ}{dx} = \frac{1}{2}\left(\frac{S_o T}{A} - \frac{gA^2}{\alpha K^2}\right) \tag{36}$$

of which K is the conveyance of the gutter, which is equal to $1.49\ A\ R^{2/3}\ n$ for English units. The left side of Eq. (36) is determined by the position of the cross section in the gutter. The discharge, Q, at a cross section x distance from the starting point is equal to the initial discharge and the added discharge along the channel $q \cdot x$. The right side of Eq. (36) is completely determined by the depth selected at a given section. Determining the right side of Eq. (36) for a number of selected depths at a given cross section will enable the elevation of the transition profile to be computed. This will be explained in the following example:

Problem: A triangular highway gutter is designed to carry a varying lateral discharge of 0.1 cfs per linear foot along the gutter. The distance between two adjacent inlets is 50 ft and the initial discharge from the upstream inlet is assumed to be equal to zero. The longitudinal slope of the gutter is 0.05, starting at an upstream bottom elevation of 5.0 ft. Compute the flow profile for the design discharge, assuming $n = 0.015$ and $\alpha = 1.0$.

Solution: The first step is to determine the control section from which the flow profile computation can be started. The computer flowchart for the position of the transition profile is given in Fig. 4 and a complete Fortran program listing appears in Table 1 for triangular, rectangular, and trapezoidal gutter sections. The computer output of this computation is given in Table 2. This enables the transition profile and critical depth line to be plotted on the same longitudinal scale as shown in Fig. 5. At a given distance from the inlet the discharge, Q, (column 5) is obtained by multiplying the distance by the varying inflow. Using the general critical flow criteria, the critical depth corresponding to the computed inflow discharge, Q, is obtained. The computer continues to calculate Q and D_c for different values of D until the condition that $D = D_c$ is satisfied. This will determine the distance, x, at which critical flow condition takes place. The results are shown graphically in Fig. 5. The intersection of the D - x curve with the D_c - x curve determines the location of the critical section. For this problem the critical section lies 28 ft downstream from the preceding inlet; the critical depth is 1.079 ft.

3.2.4 Numerical Computions and Computer Program

The differential equation of spatially varied flow with increasing discharge is nonlinear because the right-hand side of Eq. (20) or Eq. (35) is a nonlinear function of y, the dependent variable. Owing to the fact that analytic solutions to the nonlinear differential equations are hard to obtain, the dynamic equation of spatially varied flow may only be integrated by numerical methods. The numerical integration of this equation is impractical without a high-speed digital computer. By using the techniques of numerical analysis the problems of accuracy and convergence affecting the solution can be kept under proper control. Because this problem does not need very high accuracy, for instance 10 digits after the decimal point, a simple trapezoidal integration rule is being used. The simplest case of closed integration is shown schematically in Fig. 6. The two points, $x_o = a$ and $x_1 = b$, are used to determine a first-degree polynomial y(x) or straight-line approximation of p(x). The required area under the solid curve of Fig. 6 is approximated by the area under the dotted straight line which is the shaded trapezoid. A detailed description of numerical integration process using trapezoidal rule is given in Fig. 7, which can be used to integrate any first-order linear or nonlinear differential equation. The computer approach of this integration method is as follows:

1. From known y_i obtained either as an initial value or from a previous calculation $(dy/dx)_i$ can be computed from Eq. (23) or Eq. (38).

2. Assume $(dy/dx)_{i+1} = (dy/dx)_i$ as a first approximation.

3. Calculate an approximate y_{i+1} from

$$y_{i+1} = y_i + \frac{(dy/dx)_{i+1} + (dy/dx)_i}{2} \Delta x \tag{37}$$

Using the $(dy/dx)_{i+1}$ obtained from steps 2 or 4, Δx is the integration step along the gutter. If Δx is chosen small enough, then the error introduced by the assumption that dy/dx varies linearly between the Δx integration distance can be neglected.

4. Compute a new $(dy/dx)_{i+1}$ from Eq. (20) or Eq. (35) using the approximate y_{i+1} obtained in step 3.

5. Repeat steps 3 through 5 until the new $(dy/dx)_{i+1}$ in step 4 is very close to the previous assumed value in step 2. How close they should be is predetermined by the user. After this requirement is met then the whole procedure is advanced by one integration step Δx.

The Frotran IV computer flowchart and program are given in Fig. 7 and Table 3, respectively. The final result of water surface profile is listed in Table 4.

3.3 Theory of Flow into Drop Inlets

3.3.1 Types of Drainage Inlets

After the design discharge for a drainage inlet has been determined, as through an hydrological study, the hydraulic design of the inlet can be made; the first step in the design is studying the hydraulic characteristics of the inlets.

Generally speaking, there are three types of inlets that are commonly used in the United States: Side, drop, and combined inlets. A side inlet is placed at the side of the curb with an adjacent grating in the bottom of the gutter; it has been used without a grating in order to increase its efficiency. The second type, the drop inlet, is placed at relatively flat longitudinal angles, in side channels of the highway. This type of inlet is commonly covered by a grate through which the water falls. The grates not only serve to strengthen the structure of the inlet, but also serve to prevent possible damage to traffic vehicles. The third type is a combined inlet, the side and deop inlets being placed together.

One of the common methods used to enable water to flow more easily into an inlet is to lower the gutter at the upstream end of the inlet and to raise it at the downstream end thus enabling water to flow more readily into the inlet; such units are commonly termed depressed inlets. However, a deep depression is objectionable, because of the damage that might occur to a vehicle. It is doubtful whether a gutter depression is the best way to increase the inlet efficiency. Tests made at Johns Hopkins University showed that an increase in depression length from 4 ft to 6 ft gave an increase in the capacity of only 25% for a street grade of 4%, a transverse slope of 1:18, and a depression depth of 2.5 in. Many factors implicit in a depressed inlet, such as, the length, width, depth, and the condition of the approaching flow, which affect the performance of the depressed inlet, make a theoretical study even more involved. A study of depressed inlets will not be included herein.

3.3.2 Simplified Method

Assume a uniform velocity distribution in highway gutter flow. Considering a freely falling stream of water in front of a drop inlet, as shown in Fig. 7, the following two equations must be satisfied

$$L_o = V_o t \text{ , and} \tag{38}$$

$$y_o = gt^2/2 \tag{39}$$

By eliminating time, t, in the above two equations the length, L_o, can be obtained as

$$L_o = V_o \sqrt{\frac{2y_o}{g}} \tag{40}$$

where L_o is the theoretical length of the inlet required to catch the entire flow, ft,

V_o is the average velocity of flow corresponding to L_o, fps, and

y_o is the depth of flow in the gutter corresponding to L_o, ft.

If the actual length of the inlet, L, is less than, L_o, then the depth of flow, y, which can be trapped into the inlet is given by

$$y = y_o \left(\frac{L}{L_o}\right)^2 \tag{41}$$

which is obtained by substituting L for L_o and y for y_o into Eq. (40) and dividing with Eq. (40). If both the velocity and the depth of flow in the gutter are known, the ideal length of the inlet, L_o, can be computed from Eq. (40). If the inlet has a length, L, smaller than the ideal length, L_o, not all the water flowing the gutter will be caught by the inlet. The depth of water caught can be obtained from Eq. (41). In highway gutter flow, see Fig. 4b, the discharge into the inlet, Q, can be obtained as follows:

Inasmuch as the average velocity, V_o, and the depth of water in the gutter, y_o, are known, the theoretical length of the inlet, L_o, from Eq. (40) can be computed

$$L_o = V_o \sqrt{\frac{2y_o}{g}} \tag{42}$$

The water depth, y, at a distance, x, from the curb is given by

$$y = y_o - z/\tan\theta_o \tag{43}$$

where θ_o is the angle between the bottom of the gutter and the vertical. Substituting Eqs. (42) and (43) into Eq. (41), yields

$$y_o - z/\tan\theta_o = y_o\left(L/V_o \frac{2y_o}{g}\right)^2 \tag{44}$$

The continuity equation for flow into an inlet can be shown as:

$$Q = \int_o^{y_o \tan\theta_o} V_o\, y dz \text{ or} \tag{45}$$

$$Q = \int_o^{y_o \tan\theta_o} V_o\, (y_o - z/\tan\theta_o)\, dz \tag{46}$$

using Eq. (43). Sutstituting Eq. (44) into Eq. (46) and integrating, leads to

$$Q = \int_o^{y_o \tan\theta_o} V_o\, y_o\, (L/L_o)^2\, dz \text{ , or}$$

$$Q = gL^2\, y_o \tan\theta_o/2V_o \tag{47}$$

It can be readily seen that for given highway gutter and inlet dimensions the only unknown which needs to be computed in Eq. (47) is discharge Q. This will enable us to obtain the theoretical capacity of an inlet under a given flowing condition.

The above approach to this problem has several assumptions underlying it which simplify the mathematical derivation. Additionally, the assumptions cause the problem to deviate from reality. A uniform velocity distribution in the highway gutter rarely occurs in the field. Furthermore, the freely falling stream of water in front of the drop inlet is another approximation, because only the water at the free surface can be considered as undergoing a free-falling action. The water particles within the stream itself mutually affect each other. The free-falling assumption also implies that the size of the pipe connected to the inlet is large enough to drain readily all the water flowing into the inlet. If the size is inadequate so that flow into the inlet is more than the flow out, then an elevation head is formed above the entrance to the pipe; an elevation head at the pipe would obviate the free-falling assumption. Consequently, the using of Eq. (47) should be used very carefully when applied to field conditions.

3.3.3 Specific Energy Method

The flow in a highway gutter with drop inlets is a case of spatially varied flow with decreasing discharge. Assume a uniform velocity distribution in highway gutter flow, $\alpha = 1$, and the longitudinal slope is very small, i.e. $\theta \cong 0$. Considering a freely falling stream of water at an inlet, as shown in Fig. 7, the specific energy at any section of the gutter is

$$E = y + \frac{V^2}{2g} = y + \frac{2Q^2}{gy^4 \tan^2 \theta_o} \qquad (48)$$

From a previous investigation Giorgio (11) concludes that for spatially varied flow with decreasing discharge, the specific energy can be considered constant along the gutter, or $\frac{dE}{dx} = 0$. Setting the derivation of E of Eq. (48) with respect to x equal to zero and rearranging terms, leads to

$$\frac{dy}{dx} = \frac{4Qy\left(-\frac{dQ}{dx}\right)}{gy^5 \tan^2 \theta_o - 8Q^2} \tag{49}$$

in which dy/dx is the water surface profile, and

- dQ/dx is the discharge withdrawn through a **length, dx,** of the inlet.

If either dy/dx or dQ/dx is known, the other can be computed by means of the above general dynamic equation for the flow under consideration. Under the usual condition of flowing the discharge through a length, dx, of the inlet may be expressed as [Ref. (28)]

$$-\frac{dQ}{dx} = JC_d y \tan \theta_o \sqrt{2gE} \tag{50}$$

in which J is the ratio of the open area to the total area of the inlet, and

C_d is the discharge coefficient of the inlet.

From Eq. (48) the discharge can be expressed as

$$Q = y^2 \tan \theta_o \sqrt{\frac{g}{2}(E-y)} \tag{51}$$

Substituting Eq. (50) and Eq. (51) into Eq. (49) and simplifying results in

$$\frac{dy}{dx} = \frac{4JC_d\,E(E-y)}{5y - 4E} \tag{52}$$

Equation (52) may be integrated either by a numerical method or by an exact solution. The exact solution for Eq. (52) can be found in any book having a table of integrals (25), which reads

$$x = \frac{2E - 5y}{6JC_d} \sqrt{1 - \frac{y}{E}} + C_1 \tag{53}$$

The integration constant C_1 is determined at the point, where $x = 0$ and $y = D_o$, as $C_1 = - \frac{2E - 5D}{6JC_d} \; 1 - \frac{D}{E}$. Thus, Eq. (56) can be rewritten as

$$x = \frac{1}{6JC_d} \left[(2E - 5y) \sqrt{1 - \frac{y}{E}} - (2E - 5D_o) \sqrt{1 - \frac{D_o}{E}} \right] \tag{54}$$

If we let y in Eq. (54) be equal to zero, then x will be the theoretical length required for the complete withdrawal of the flow in the gutter, or

$$L_o = \frac{1}{6JC} \left[2E - (2E - 5D_o) \sqrt{1 - \frac{D_o}{E}} \right] \tag{55}$$

In the above equation J and C_d are determined by the characteristics of the inlet in the highway gutter, and E and D_o are governed by the flow approaching the inlet. For a given inlet and a given flow condition in the gutter the theoretical length of the inlet is completely determined by Eq. (55). The following problem illustrates the method under consideration.

It is assumed that $J = 0.4$, $C = 0.60$, and $D_o = 1.0$ ft. The average velocity in the gutter is 4.01 fps and the transverse slope of the highway shoulder is 2°. The theoretical length of the inlet is to be computed so as to have a complete withdrawal of the flow in the gutter. From Eq. (48) the specific energy at the upstream end of the drop inlet is

$$E = D_o + \frac{V^2}{2g} = 1.25 \text{ ft}$$

The theoretical length of the inlet can be computed from Eq. (55) as

$$L_o = \frac{1}{6JC_d} 2E - \left[2E - (2E - 5D_o)\sqrt{1 - \frac{D_o}{E}}\right] = 2.54 \text{ ft}$$

It is also possible to calculate the theoretical width and length at any section of the inlet through Eq. (54), or

$$x = \frac{1}{1.44}\left[(2.5 - 5y)\sqrt{1 - \frac{y}{1.25}} + 1.12\right]$$

Knowing the distance, x, enables one to compute the corresponding depth, y, which, when combined with the transverse slope, θ_o, gives the theoretical width of the inlet at that specific distance. The results are listed in Table 5.

3.4 Theory of Flow into Side Inlets

3.4.1 Simplified Method

The theories of the flow into a side inlet are essentially similar to the flow into a drop inlet, see Fig. 8. Flow into a side inlet is produced by the gravitational component in the direction of flow, provided the frictional force of the gutter is neglected

$$a = g \cos \theta_o \tag{56}$$

However, for the drop inlet the driving force is the gravitational acceleration g. The width of flow shown in Fig. 9 is given by

$$T = y_o \tan \theta_o \tag{57}$$

Thus, substituting T and $g \cos \theta_o$ for y_o and g, respectively, in Eq. (40), the following relationship would hold for side inlets

$$L_o = V_o \sqrt{\frac{2y_o \tan \theta_o}{g \cos \theta_o}} \tag{58}$$

where L_o and V_o are defined the same as in the case of the drop inlet in Section 3.3.2. The discharge in the gutter is given by

$$Q_o = \frac{1}{2} V_o y_o^2 \tan \theta_o \tag{59}$$

Substituting V_o from Eq. (58) into Eq. (59) and rearranging terms lead to

$$\frac{\theta_o}{L_o y_o \quad gy_o} = \sqrt{\frac{\sin \theta_o}{8}} \tag{60}$$

For highway gutters used in practice transverse grades are very small; thus, sin θ_o is nearly equal to unity. Hence, the following maximal number can be written for the dimensionless parameter

$$\frac{\theta_o}{L_o\ y_o\sqrt{gy_o}} = \frac{1}{\sqrt{8}} = 0.35 \tag{61}$$

In the developement of the above equation a few assumptions have been made; that is, a uniform velocity distribution is considered to be present, and the effects of both longitudinal slope and friction force are neglected. Therefore, the number, 0.35, can be considered only as an ideal number value of this ratio. For practical purposes an empirical evaluation of this parameter must be made Experiments conducted by Li (20) have shown that this parameter is roughly a constant, or

$$\frac{\theta_o}{L_o\ y_o\sqrt{gy_o}} = K \tag{62}$$

where K = 0.20 for tan θ_o = 24 and 48, which correspond to transverse slopes of 1/2 and 1/4 inch per foot, respectively; for tan θ_o = 12, corresponding to a transverse slope of 1 inch per foot, K = 0.23.

If the actual length of the side opening, L, is less than the ideal length, L_o, the following relationship holds, which is similar to Eq. (41)

$$b = y_o \tan\theta_o \left(\frac{L}{L_o}\right)^a \tag{63}$$

where b is the width of flow discharging to an inlet of length, L, as shown in Fig. 9. The area of flow for a width, b, in the gutter is:

$$A = y_o b - b^2/2\tan\theta_o \tag{64}$$

Hence, the discharge, Q, flowing into the inlet is:

$$Q = (y_o b - b^2/2\tan\theta_o) V_o \tag{65}$$

Because the ideal discharge is given by Eq. (59), the ratio of actual discharge to the ideal discharge is

$$\frac{Q}{Q_o} = \frac{y_o b - \dfrac{b^2}{2\tan\theta_o}}{\dfrac{y_o^2 \tan\theta_o}{2}} = \frac{1 - \dfrac{1}{2}\dfrac{b}{y_o \tan\theta_o}}{\dfrac{1}{2}\dfrac{y_o \tan\theta_o}{b}} = 2\left(\frac{L}{L_o}\right)^2 - \left(\frac{L}{L_o}\right)^4 \tag{66}$$

using Eq. (63) as well.

For most of the field cases the actual length of the inlet is longer than 60% of the ideal length, that is, $\frac{L}{L_o} = 0.6$. The above equation, then, can be approximated by the following relationship without introducing serious error*, (Fig. 9)

$$\frac{Q}{Q_o} = \frac{L}{L_o} \tag{67}$$

---

*The argument of the reality of the simplified approach to the drop inlet in Section 3.3.2 is also applicable to this section because of the essential similarity between the two sections.

This equation implies that as long as the actual length is greater than 0.6 of the ideal length, the capacity of the side inlets is proportional to the length of the inlet. Substituting this relationship into Eq. (62), yields

$$\frac{\theta}{L y_o \sqrt{g y_o}} = K \tag{68}$$

The factor, K, for different slopes was determined experimentally by Li (20) and was previously discussed in regard to Eq. (62) in this section.

3.4.2 Specific Energy Method

The hydraulics of side inlets can be treated as a special case of spatially varied flow with decreasing discharge if one considers the water flowing through the side inlet as the decreasing discharge for the gutter. It has been concluded by Frazer (10) that five different flowing conditions can be produced at the side inlet using the critical water depth as a criterion. The flow before that water reaches the side inlet might be subcritical, supercritical, or approximately critical. Each category has its own hydraulic characteristics and should be analyzed accordingly. In the conventional analysis it is assumed that the direction of the flow through the inlet is at right angles to the inlet, similar to what has been done in Section 3.3.2. Actually, most of the time the direction of overflow makes an angle with the side inlet smaller than the right angle. It is especially true for the supercritical flow condition, in which the upstream flow velocity is much higher than in subcritical flow.

It may be concluded that the higher the flow velocity in the gutter the smaller the angle will be.

The analysis of this problem, using the specific energy method, is essentially similar to the analysis applied to drop inlets. All the assumptions which have been made for Section 3.3.3 are also applicable to this study. Equation (49) is also true for this case, or

$$\frac{dy}{dx} = \frac{4Qy\ (-\ dQ/dx)}{gy^5\ \tan^2 - 8Q^2} \tag{49}$$

If the side inlet can be approximated by a weir and can be divided into infinitesimal parts without changing its weir characteristics, then the decreasing discharge along the inlet can be computed by a basic weir formula, or

$$-\ dQ = C_e\ dx\ (y - D_e)^{3/2} \tag{69}$$

in which C_e is the weir discharge coefficient, and

D_e is the height of the side inlet above the gutter.

It is assumed that the specific energy along the inlet is constant. Equation (51) is also applicable to this problem, or

$$Q = y^2\ \tan\theta_o \sqrt{\frac{g}{2}\ (E-y)} \tag{51}$$

Substituting Eq. (69) for - dQ/dx and Eq (51) for Q in Eq (49) and simplifying

$$\frac{dy}{dx} = \frac{4C_e \ (y-D_e) \sqrt{\frac{g}{2} \ (E-y) \ (y-D_o)}}{gy \ (5y - 4E)} \tag{70}$$

The exact solution of Eq. (70) is very difficult to obtain. However, a simple numerical solution to this equation would suffice for all purposes. For each x there is a corresponding depth or y which can be obtained numerically from Eq. (70). This depth so found, combined with the transverse slope, θ_o, gives the theoretical width of the inlet at that specific cross section. Carrying out the above procedures at every section of the inlet, one can determine the overall dimension of an inlet which has a theoretical 100% efficiency.

4. EXPERIMENTAL INVESTIGATION

A model for the study of highway drainage inlets is being built in the Fritz Engineering Laboratory of Lehigh University in order to determine the capacity of the inlets currently, 1970, in use along highways and in order to develop an improved inlet. After preliminary design of the model exterior plywood with appropriate coverings was used to simulate the channel upstream from the inlet. The Manning resistance coefficient for the plywood was determined by causing water to flow over a plywood bed placed into a test channel and by performing the proper calculations.

A glass channel on the first floor of the Hydraulics Laboratory (Fig. 11) was modified for this purpose. On top of the glass bed a plywood bed was installed. The slope was determined by means of a level. After several modifications the slope ranged in placed from a low of 0.95% to a high of 1.06%, but 10 consecutive feet of channel had a slope of 1.00%. The channel modified specifically for this test has a dimension of 1.5 ft wide, 18 ft long, and 3 ft deep which includes 2 ft long of transitional section before the test section. The flow enters the channel from an 8 ft deep, 2.5 ft wide, and 3 ft long head tank through a regulating gate. Water is supplied by two main pumps which have a maximal combined capacity of nearly 6.5 cfs, the water flows from a constant-head tank to the channel through an 8-inch pipe. The flow rate is controlled by an 8-inch gate valve just before the water enters the head tank. The downstream regulating gate is 1.5 ft wide and 1.0 ft long, it also was on a 1.00%

slope in order to eliminate the drawdown effect on the testing section. Water drains from the tail tank through an existing box pipe into the main sump of the laboratory.

A Venturi meter is used to measure the rate of flow. The pressure differential across the two sections was measured on a 100-inch manometer. This Venturi meter had been calibrated by means of a volumetric tank in a previous investigation. The rating equation of the meter as determined by a least-squares procedure is given by:

$$Q = 1.05\ H^{0.552} \tag{71}$$

where H is the metering differential, ft of water, and

Q is the volumetric rate of flow, cfs.

Since the Manning formula is employed for the calculation, uniform flow in the test channel has to be established. In order to obtain uniform flow a 3 ft transitional section was installed upstream from the plywood test section. In addition to that the first 6 ft of the plywood test section was used to establish uniform flow over the remaining 10 ft of the test section. The water depth was determined by using a point gage which has an accuracy of 0.001 ft. The surface waves resulting from the Permagum used to seal the bed at the two sides of the channel affected the accuracy of water depth measurements during shallow-water flow. The mean depth at any cross section was the average of measurements at the center and quarter points; all mean depths were averaged to obtain the overall mean depth of water in the channel.

Realizing that the average value of Manning n for glass can be as high as 0.010 (3), the effect of glass walls was considered important despite the fact that the water depth was relatively shallow. The effect of the glass walls was removed from the results by applying the Einstein method (8) which divided the hydraulic radius at a cross section which has different roughnesses. Two assumptions must be made in advance. The first assumption is that the entire wetted perimeter can be divided into units which correspond to the different patterns of roughness encountered. The second assumption is that each such unit of the wetted perimeter can be treated individually which means that each unit is not affected by the other parts of the perimeter. This assumption enables one to calculate the hydraulic radius, R, corresponding to each unit with its associated roughness. For each unit of the wetted perimeter, P_n, having a specific roughness, one can find the corresponding water area, A_n, simply by multiplying this wetted perimeter and the hydraulic radius R. The subscript, n, represents the unit number. The hydraulic radius associated with a specific unit can be obtained from Manning's formula, which will be explained in detail as follows:

If there are N units in the entire wetted perimeter, N + 2 equations can be obtained. Those equations include N equations having the form

$$V = f_n (S, n_n, R_n) \tag{72}$$

in which V and S are constant for all units, one equation having the form

$$P = \sum_{n=1}^{N} P_n \tag{73}$$

which states that the summation of all the wetted-perimeter units should equal the total wetted-perimeter, and one equation having the form

$$A = \sum_{n=1}^{N} A_n \tag{74}$$

which states that the summation of all the water-area units should equal the total area. The N + 2 equations enable the roughness to be computed for every unit, presuming that all the other units of roughness are known. The following numerical example will give a clearer understanding of this approach. Data used are those obtained in the test to determine the Manning n of plywood. The test channel had glass sides and a plywood bottom, a constant 1.00% slope, and a width of 1.50 ft. The length of the channel was assumed long enough to establish uniform flow in the test section. The Manning n of glass was taken as 0.010. The flow rate was 0.453 cfs and the mean water depth was 0.115 ft. The Manning n for the plywood can be computed from the above information.

1. The water area is 0.1719 sq ft.

2. The average velocity in the testing channel is 2.64 ft/sec.

3. Assuming that each unit can be treated as an individual one, the Manning formula for the glass wall is

$$V = \frac{1.486}{n_g} S^{1/2} R_g^{2/3}$$

in which n_g is the Manning n for glass, and

R_g is the hydraulic radius corresponding to glass.

The hydraulic radius for the glass is, from the above formula

$$R_g = \left(\frac{n_g}{1.486} \frac{V}{S}\right)^{3/2} = 0.0747 \text{ ft}$$

4. The water area corresponding to the glass wall is

$$A_g = 2d\ R_g = 0.0171 \text{ sq ft}$$

From Eq. (67) the water area corresponding to the plywood bed is

$$A_p = A - A_g = 0.1548 \text{ sq ft}$$

5. The hydraulic radius applicable to the plywood bed is

$$R_p = \frac{A_p}{b} = 0.103 \text{ ft}$$

in which b is the channel width.

6. Using the second assumption again, the roughness coefficient n_p for the plywood is thus obtained

$$n_p = \frac{1.486}{V} S^{1/2} R_p^{2/3} = 0.0123$$

A series of tests were made using the flow rate as the variable. All the data were analyzed according to the previous procedure, and the results are plotted in Fig. 10.

5. SUMMARY AND CONCLUSIONS

The results of the present investigation brought to light some aspects of inlet design. Several conclusions can be drawn and are summarized in the following paragraphs:

1. The Manning formula is the one most widely used to describe the friction resistance in the gutter. However, the success of its application is directly dependent upon the choice of the proper friction coefficient, which in turn depends upon the characteristics of the gutter. The Manning formula can be applied only for steady uniform flow conditions.

2. Water flowing in highway gutters is more accurately described by using the theory of spatially varied flow. Both the momentum and energy approaches were used. They gave similar differential equations (Eq. (20) and Eq. (35)). The momentum approach is a vector relationship, whereas the energy approach is an algebraic relationship of scalar quantities. However, both Eq. (20) and Eq. (35) can be used for the,numerical solution to this problem.

3. The computer program given in page 57 can be used for either rectangular, trapezoidal, or triangular shapes of highway gutters. It has a large flexibility.

4. Two different approaches to the problem of flow into drainage inlets were applied, namely the specific energy method and the simplified method. Theoretically speaking, the first method is

more sound. However, the simplified method is easier to apply to the field condition, provided the parameter k is known. Equations derived from these approaches were Eq. (47) and Eq. (54).

5. The theoretical approaches used for the drop inlets and side inlets are essentially similar. Flow into a drop inlet is produced by the gravitational force, whereas the flow flowing into the side inlets is produced by the gravitational component in the direction of flow. Equation (47) is used to describe the flow at a drop inlet and Eq. (62) for a side inlet.

6. Due to the assumptions made in the derivation of the above equations, a pure theoretical solution to this problem might not prove satisfactory in some situations. In addition to that, physical constants, such as, K, n, etc., should be evaluated. Consequently, an experimental study of particular inlet design is strongly recommended to reach accurate results.

FIGURES

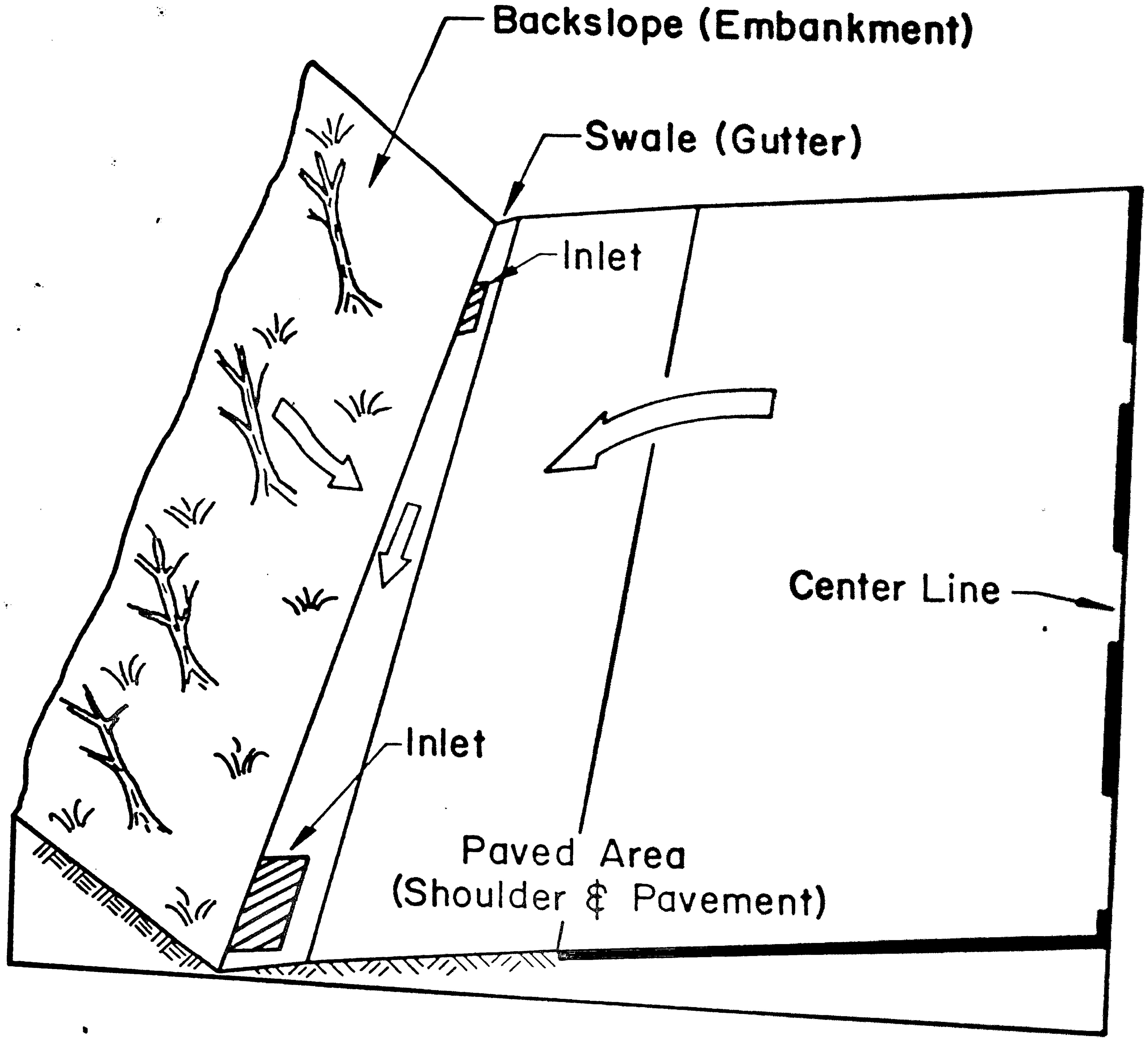

Fig. 1: General Layout for Highway Surface Drainage Inlet

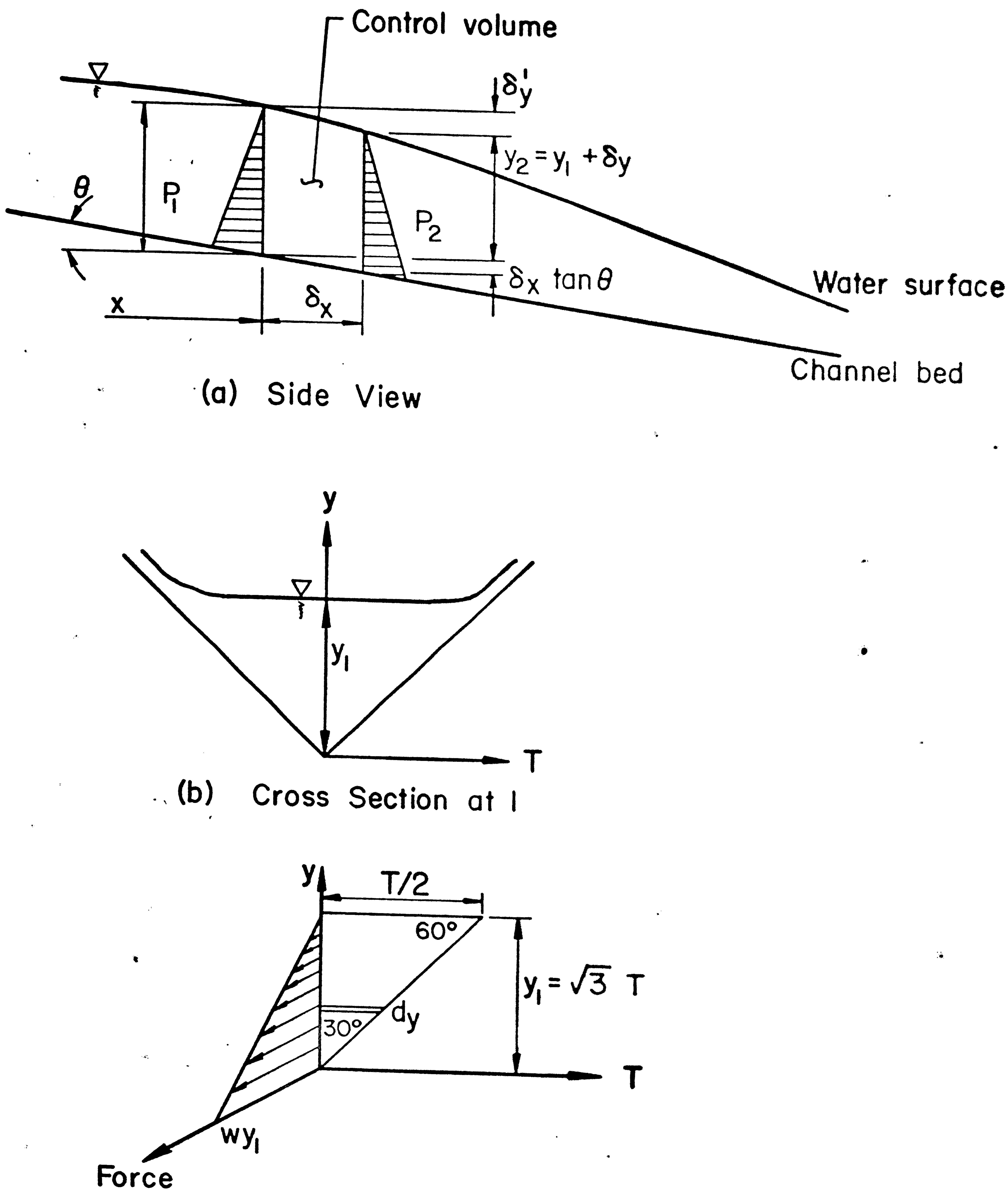

Fig. 2: Flow in a Highway Gutter: Momentum Approach

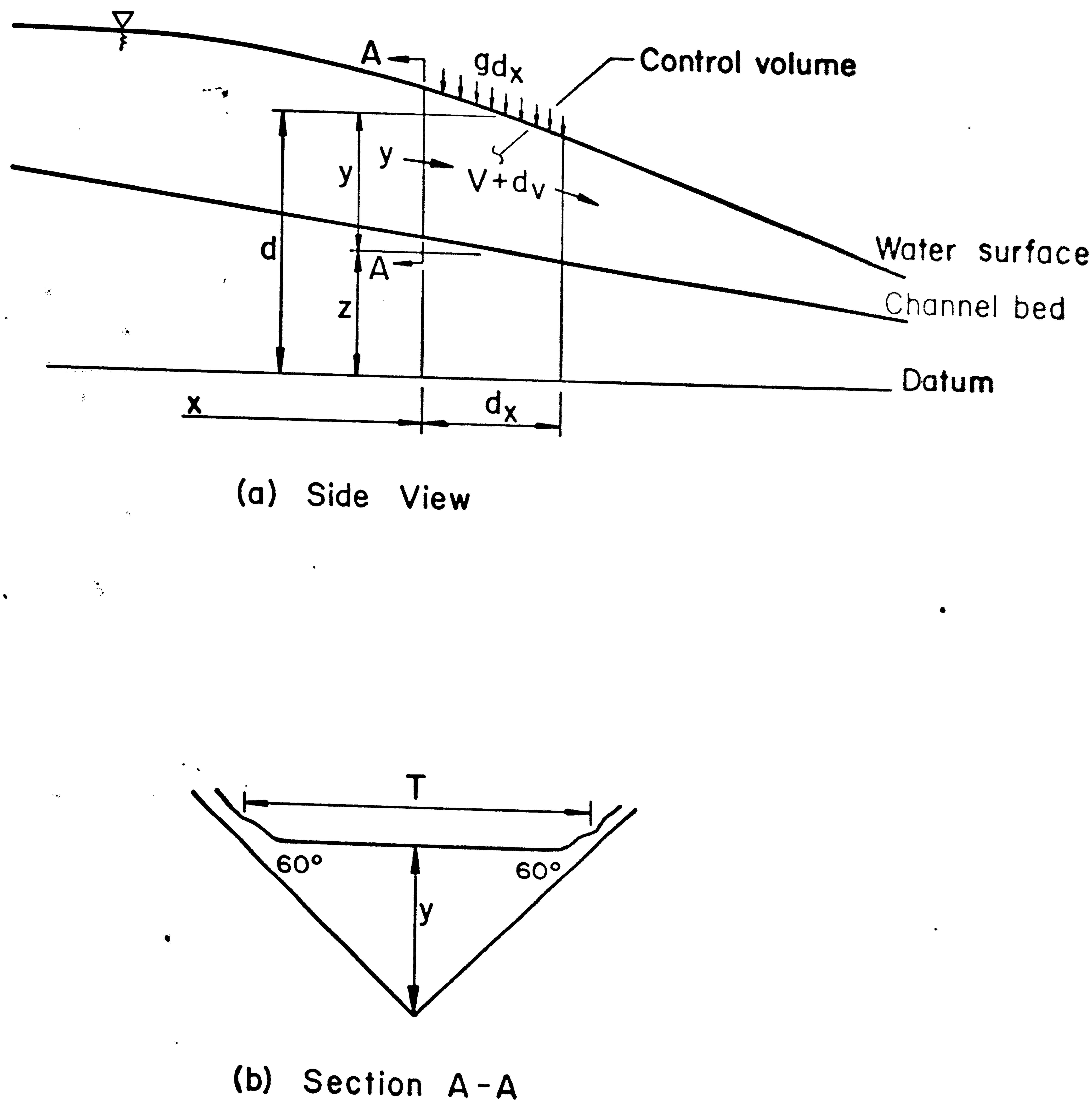

Fig. 3: Flow in a Highway Gutter: **Energy Approach**

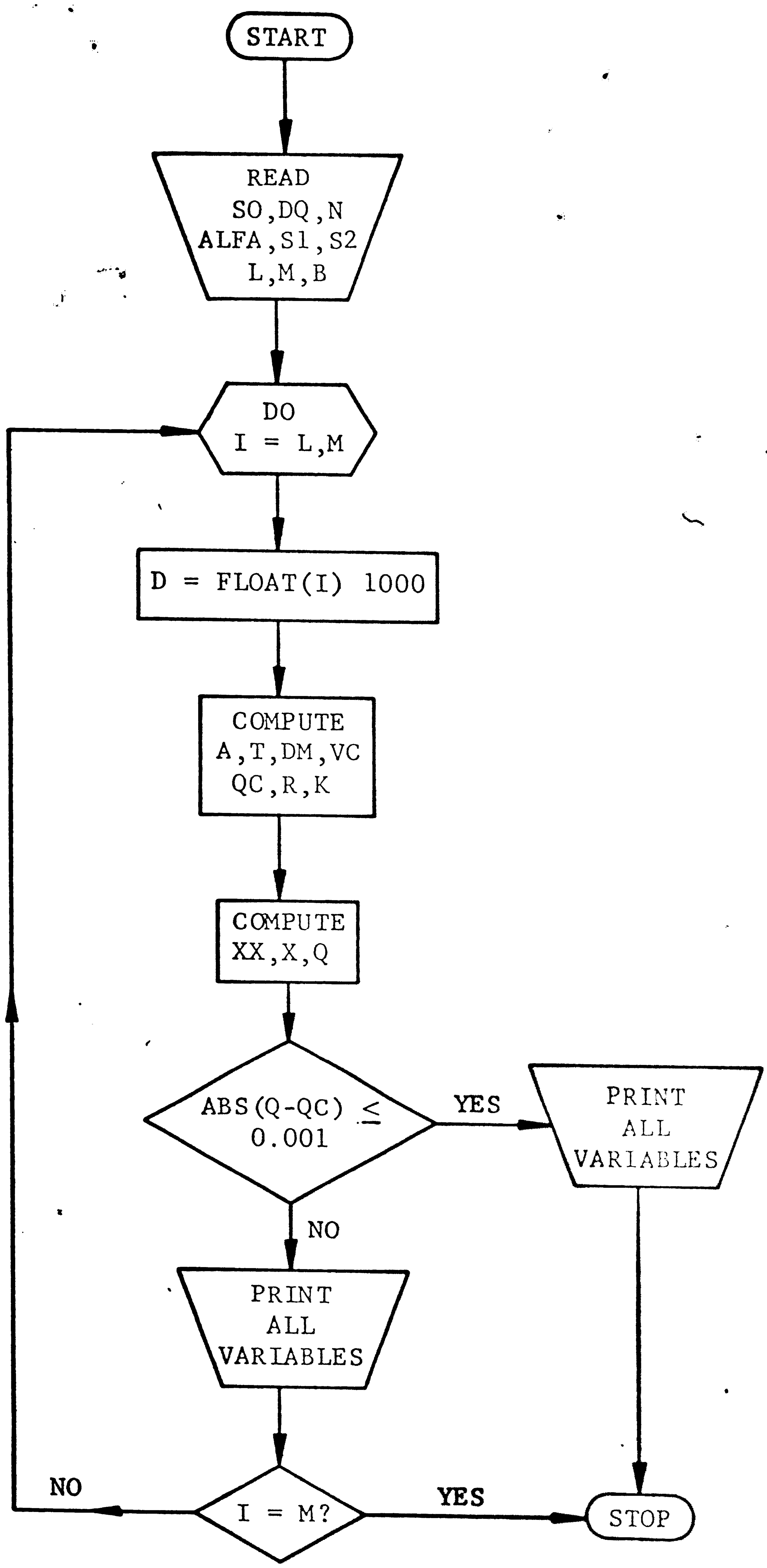

Fig. 4: Computer Flowchart for Calculation of Control Point

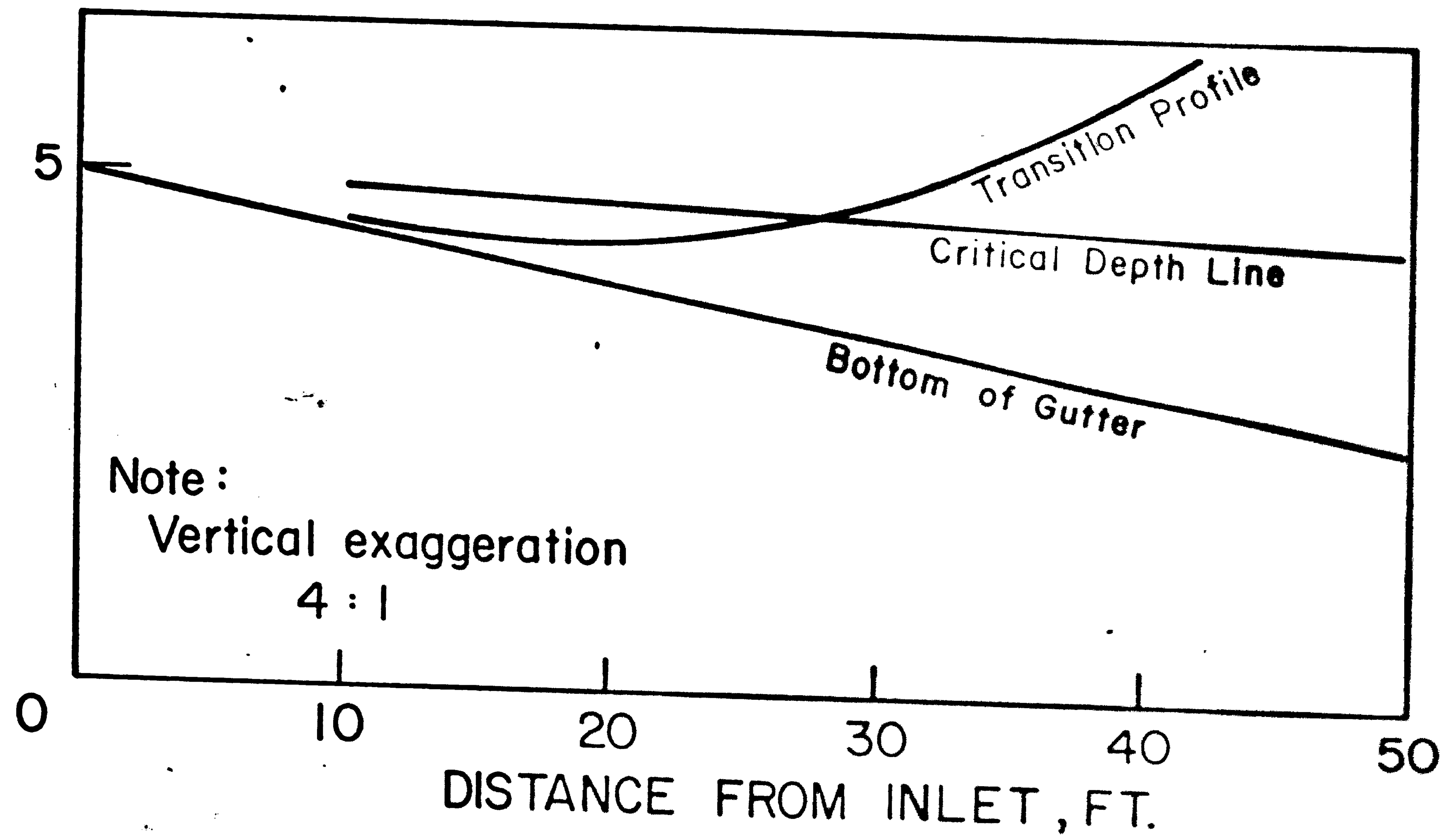

Fig. 5: Computer Solution for the Determination of Critical Section

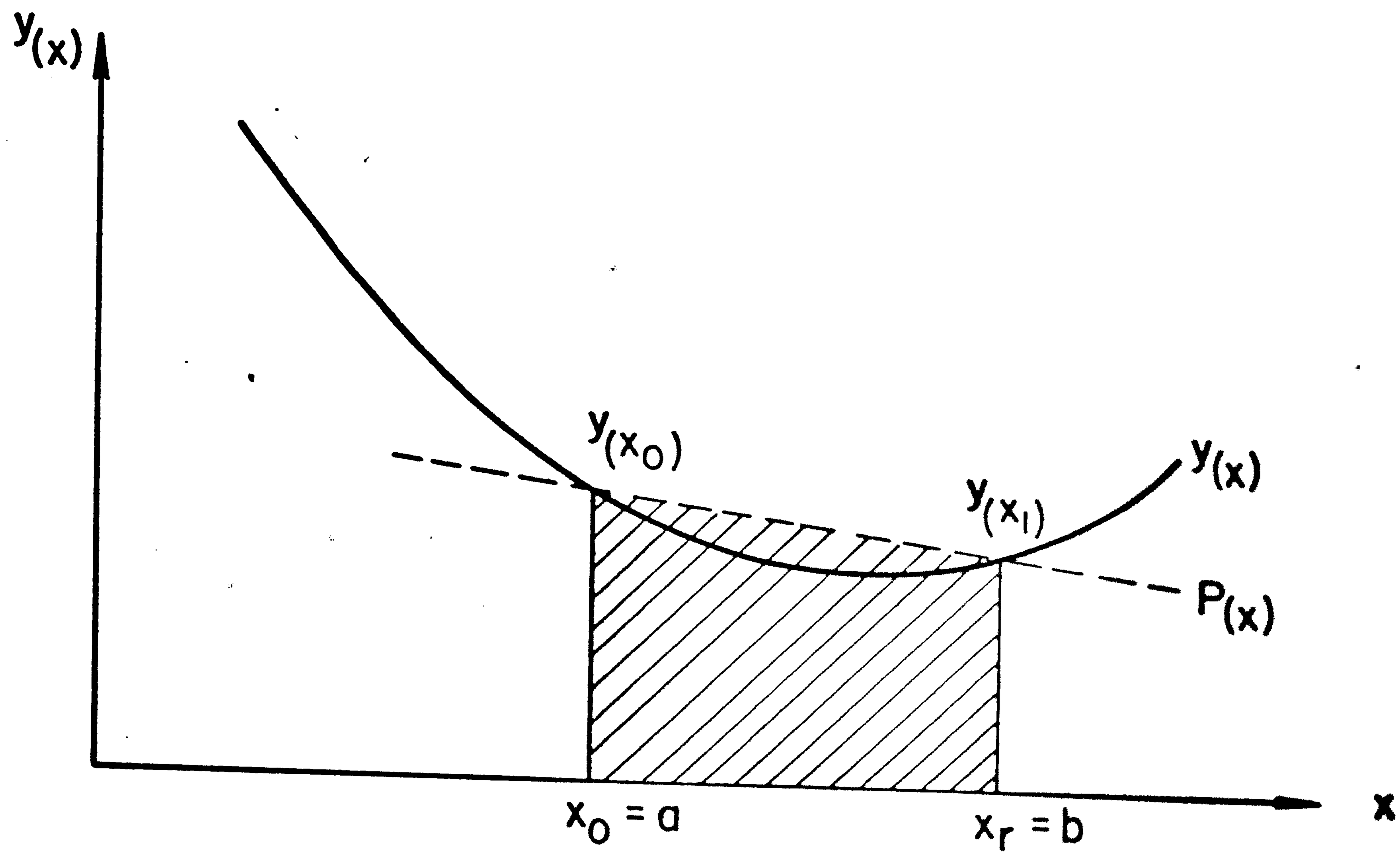

Fig. 6: The Trapezoidal Rule of Numerical Integration

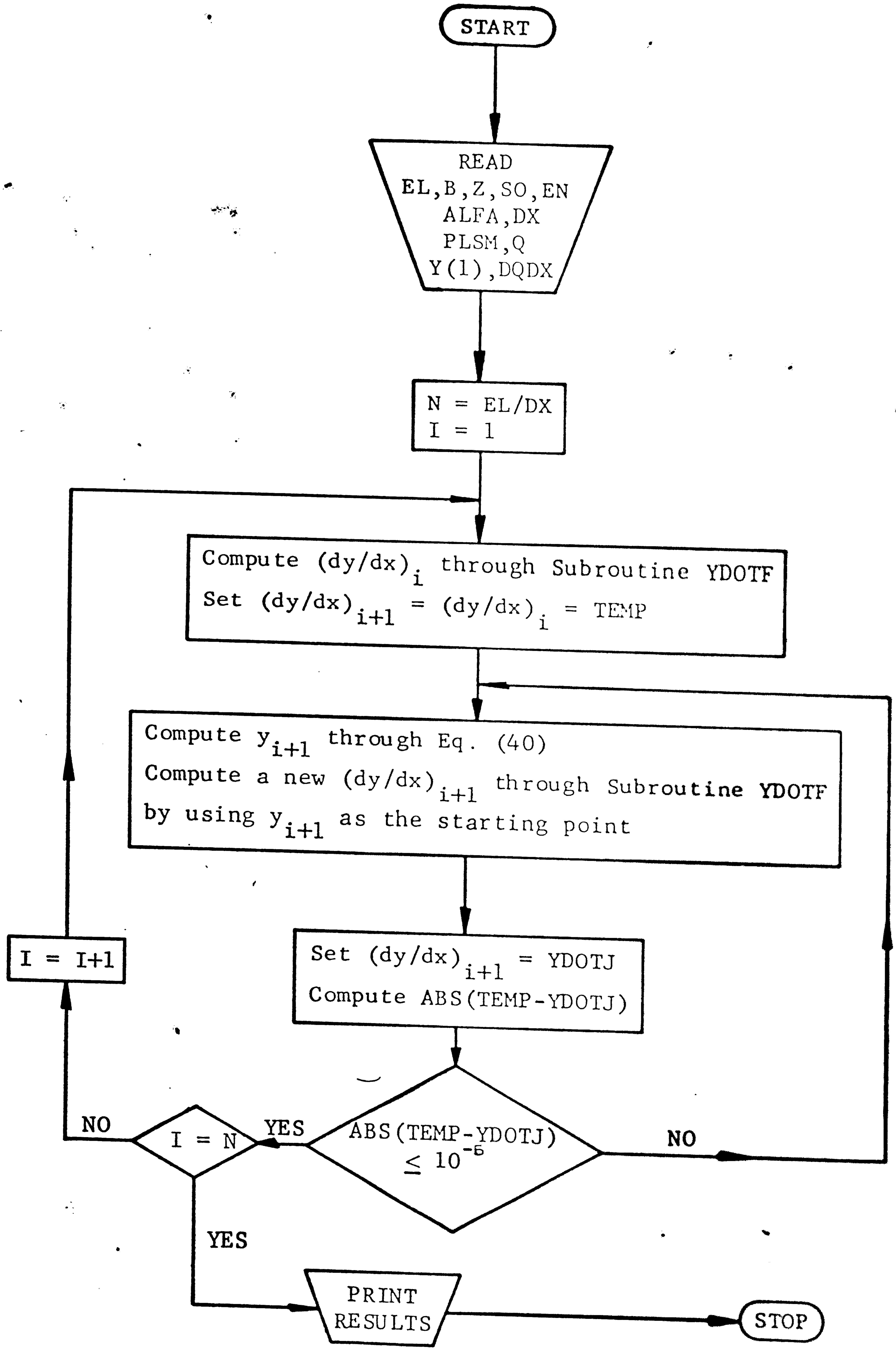

Fig. 7: Computer Flowchart for Calculation of Water Surface

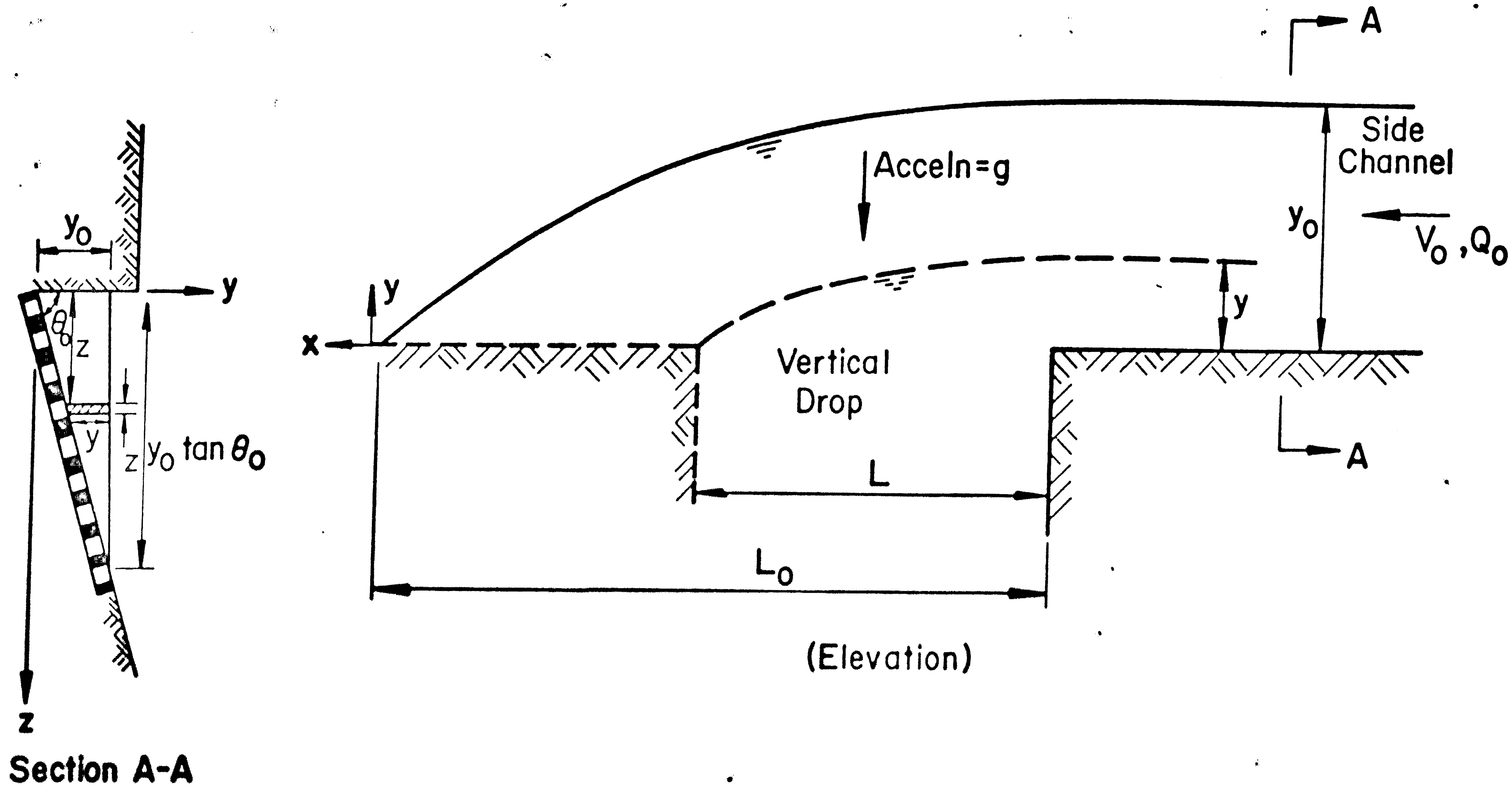

Fig. 8: Drop at the End of a Channel

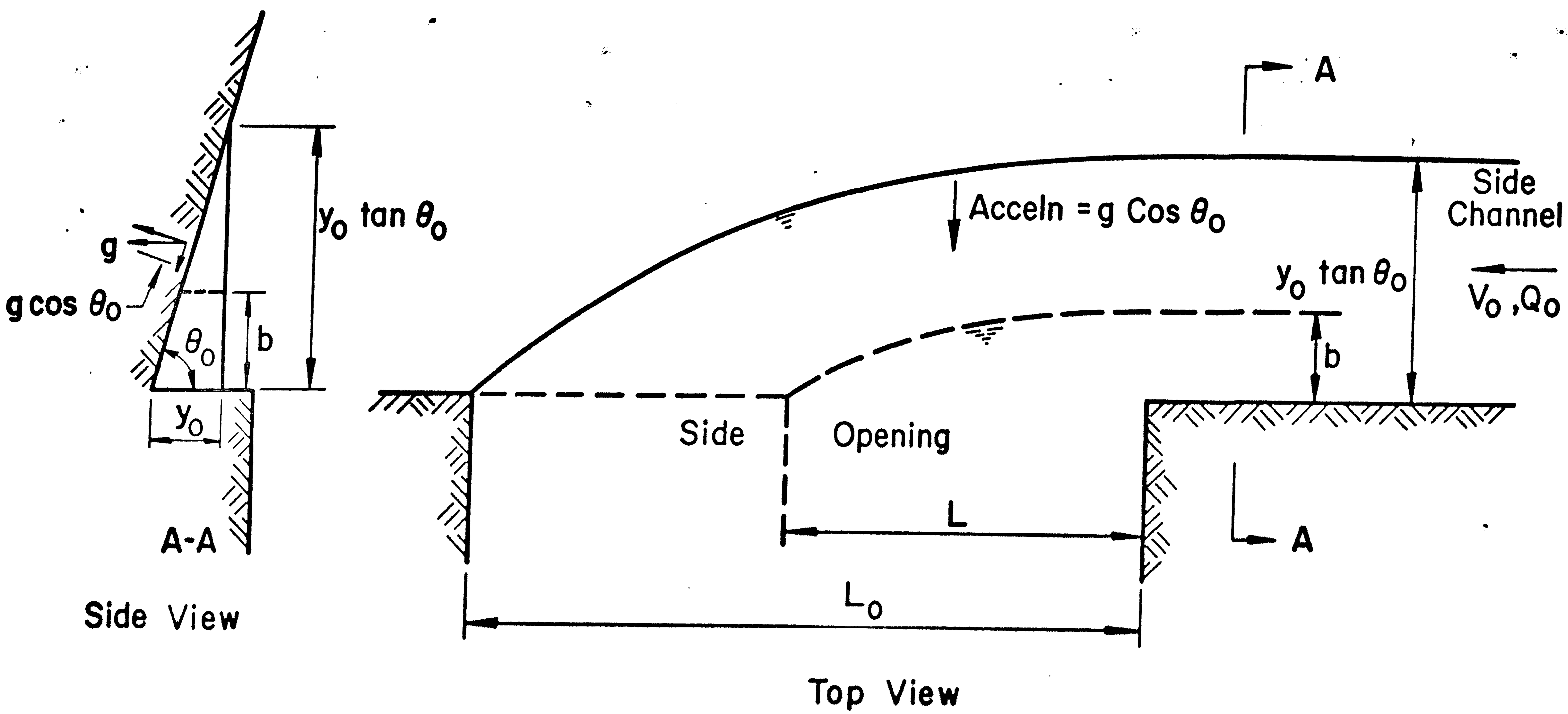

Fig. 9: Side Inlet without Grate

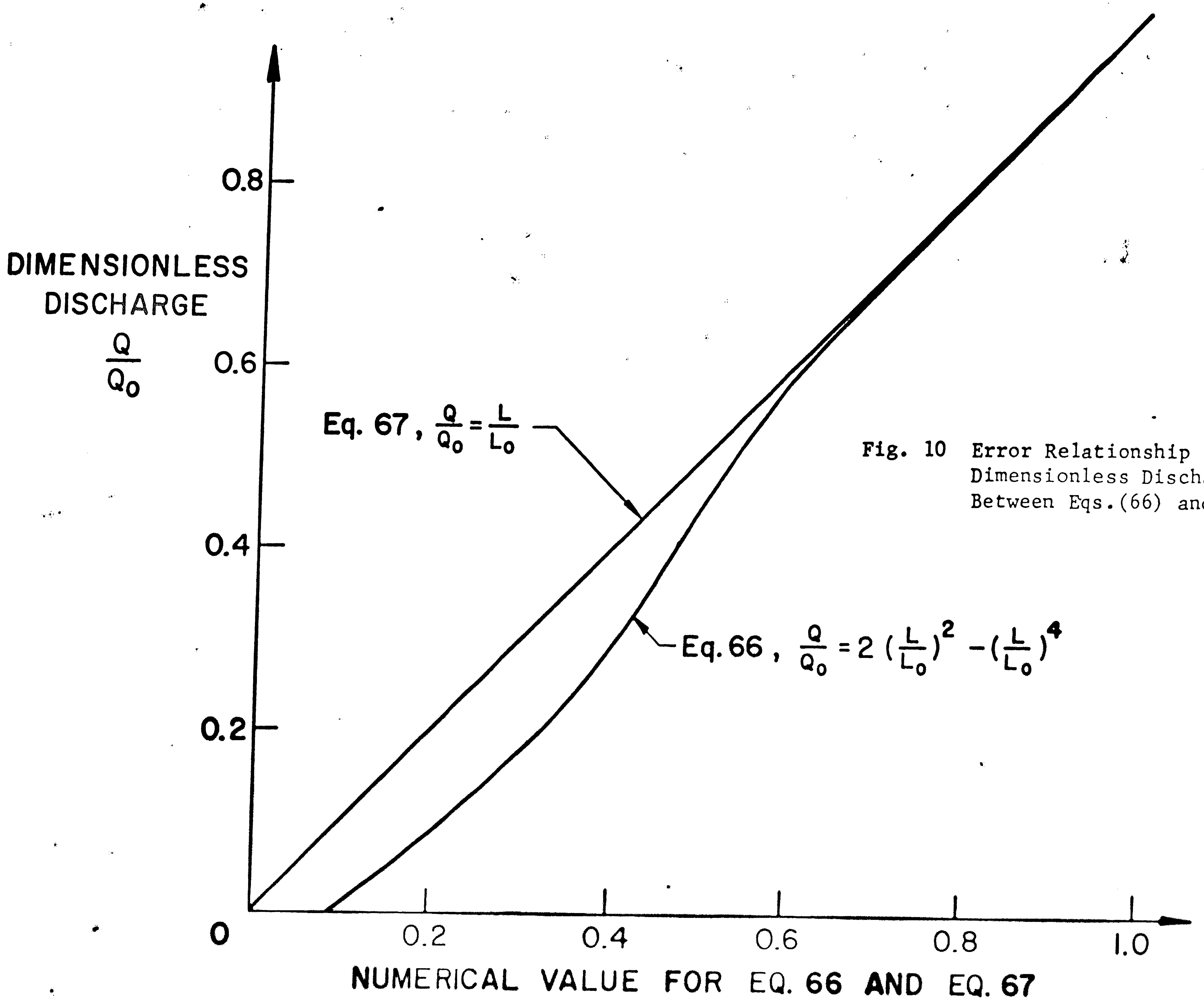

Fig. 10 Error Relationship for Dimensionless Discharge Between Eqs.(66) and (67)

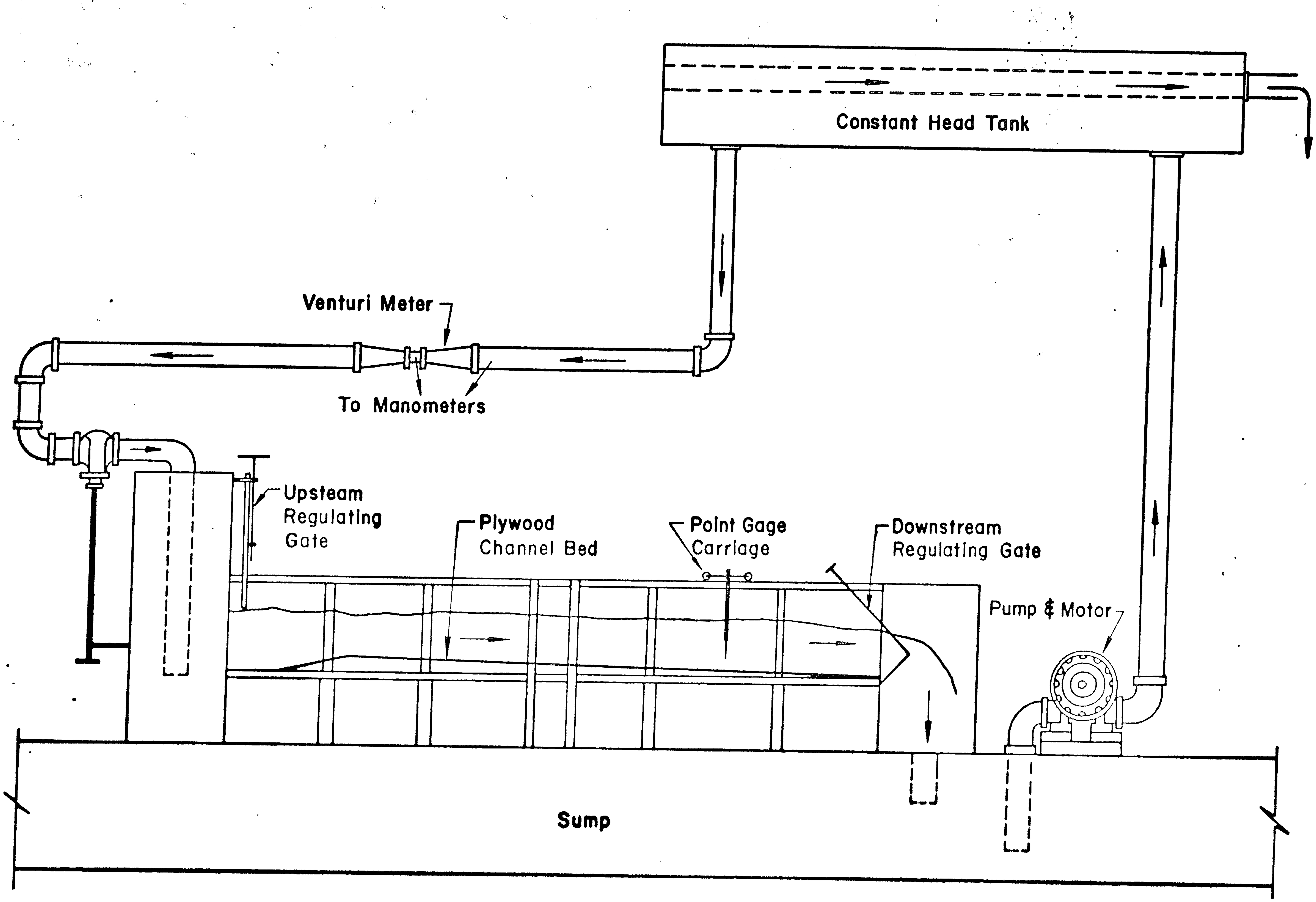

Fig. 11: General Layout of Glass Wall Flume with Plywood Bed

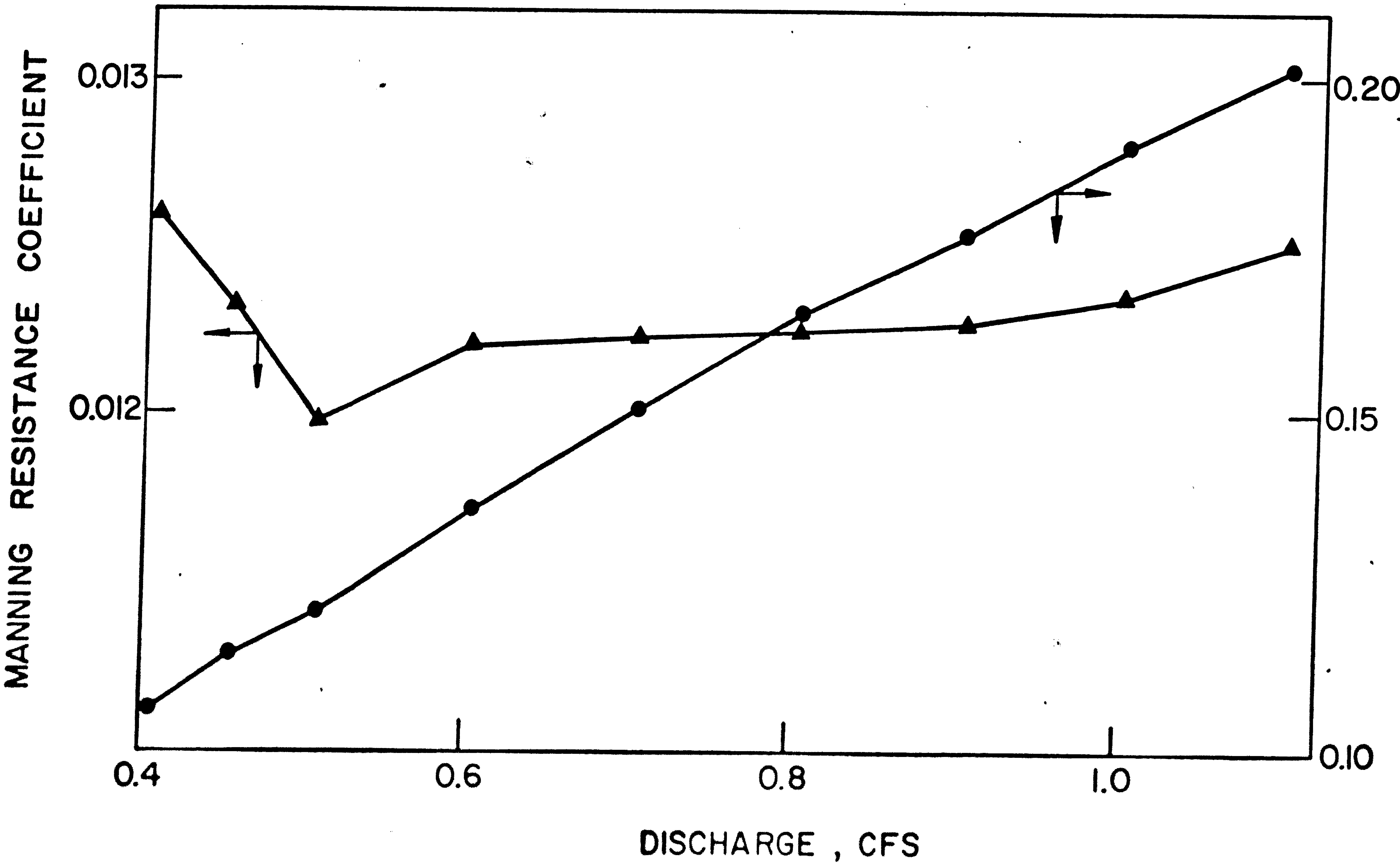

Fig. 12 Manning n Measurement for the **Exterior Plywood**

TABLES

Table 1: **Computer Program of Computing Transition Profile and Critical Depth**

```
              PROGRAM DECOP(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)
000003        REAL K,N
        C
        C     THIS IS A FORTRAN IV PROGRAM TO DETERIMINE THE CONTROL
        C     POINT FOR RECTANGULAR, TRAPEZOIDAL, AND TRIANGULAR
        C     HIGHWAY GUTTERS
        C     ***************************************************
        C     *THE FOLLOWING VARIABLE NAMES ARE USED IN THIS PROBLEM*
        C     ***************************************************
        C     SO= LONGITUDINAL SLOPE OF THE GUTTER, IN FEET PER FOOT
        C     DQ= THE VARYING DISCHARGE ALONG THE GUTTER, IN CFS PER
        C         FOOT
        C     B= BREADTH OF CHANNEL BOTTOM, IN FEET
        C     S1,S2= SIDE SLOPES OF CHANNEL, IN FEET PER FOOT
        C     D= WATER DEPTH, IN FEET
        C     A= CROSS-SECTIONAL AREA OF CHANNEL AT DEPTH D, IN
        C        SQUARE FEET
        C     T= WATER SURFACE WIDTH, IN FEET
        C     DM= HYDRAULIC DEPTH, IN FEET
        C     VC= CRITICAL VELOCITY, IN FEET PER SECOND
        C     EN= MANNINGS ROUGHNESS COEFFICIENT,N, FOR GUTTER
        C     QC= CRITICAL DISCHARGE, IN CFS
        C     ALFA= ENERGY COEFFICIENT
        C     R= HYDRAULIC RADIUS, IN FEET
        C     K= CONVEYANCE OF A CHANNEL OR 1.49*A*(R**(2/3))/N
        C     XX= 1/2*(DQ/DX) OR (SO*T/A-G*A*A/ALFA*K*K)/2
        C     X= DISTANCE MEASURED FROM THE END OF THE FIRST INLET
        C        ALONG THE GUTTER, IN FEET
        C     Q= ACTUAL DISCHARGE, IN CFS
        C     DC= CRITICAL DEPTH, IN FEET
        C
```

Table 1: **Computer Program of Computing Transition Profile and Critical Depth (continued)**

```
000003          WRITE(2,10)
000007          READ (1,20) SO, DQ, N, ALFA, S1, S2, B, L, M
000035          DO 100 I=L,M
000037          D=FLOAT(I)/1000.
000041          T= B+(S1+S2)*D
000045          A= T*D/2.0
000047          QC= SQRT((D**5.0*32.2*(S1+S2)*(S1+S2))/8.0)
000061          R=A/(2.0*T)
000064          K=(1.49*A*R**0.6667)/N
000072          XX=((SO*T/A)-(32.2*A*A/(ALFA*K*K)))/2.0
000101          X=1.0/XX
000103          Q=X*DQ
000105          T5=8.0*Q*Q/((S1+S2)*(S1+S2)*32.2)
000112          DC=(T5)**0.2
000115          WRITE(2,30) D,R,K,X,Q,DC
000135          IF (ABS(QC-Q)-0.001) 101,101,100
000142    10    FORMAT(1H1,5(/),21X,1HD,9X,1HR,7X,1HK,8X,1HX,9X,1HQ,7X,2HDC,//)
000142    20    FORMAT(7F10.4,2I5)
000142    30    FORMAT(16X,6F9.4)
000142   100    CONTINUE
          C     ****************************************
          C     *THE FOLLOWING DATA ARE USED FOR THIS PROBLEM*
          C     ****************************************
          C     SO=0.05 , DQ=0.1 , N=0.016 , ALFA=1.0 , S1=0.5774
          C     S2=0.5774 , L=1040 , M=2000
000145   101    CALL EXIT
```

Table 2: Output of Computer Program (Table 1)

D	R	K	X	Q	DC
1.0400	.2600	23.6904	27.1087	2.7109	1.0648
1.0410	.2602	23.7512	27.1321	2.7132	1.0652
1.0420	.2605	23.8121	27.1555	2.7156	1.0656
1.0430	.2607	23.8731	27.1790	2.7179	1.0660
1.0440	.2610	23.9342	27.2024	2.7202	1.0663
1.0450	.2612	23.9953	27.2258	2.7226	1.0667
1.0460	.2615	24.0566	27.2493	2.7249	1.0671
1.0470	.2617	24.1180	27.2727	2.7273	1.0674
1.0480	.2620	24.1795	27.2961	2.7296	1.0678
1.0490	.2622	24.2411	27.3195	2.7320	1.0682
1.0500	.2625	24.3027	27.3429	2.7343	1.0685
1.0510	.2627	24.3645	27.3664	2.7366	1.0689
1.0520	.2630	24.4264	27.3898	2.7390	1.0693
1.0530	.2632	24.4883	27.4132	2.7413	1.0696
1.0540	.2635	24.5504	27.4366	2.7437	1.0700
1.0550	.2637	24.6126	27.4600	2.7460	1.0703
1.0560	.2640	24.6748	27.4834	2.7483	1.0707
1.0570	.2642	24.7372	27.5068	2.7507	1.0711
1.0580	.2645	24.7997	27.5303	2.7530	1.0714
1.0590	.2647	24.8622	27.5537	2.7554	1.0718
1.0600	.2650	24.9249	27.5771	2.7577	1.0722
1.0610	.2652	24.9876	27.6005	2.7600	1.0725
1.0620	.2655	25.0505	27.6239	2.7624	1.0729
1.0630	.2657	25.1134	27.6473	2.7647	1.0733
1.0640	.2660	25.1765	27.6707	2.7671	1.0736
1.0650	.2662	25.2396	27.6941	2.7694	1.0740
1.0660	.2665	25.3029	27.7175	2.7717	1.0743
1.0670	.2667	25.3662	27.7409	2.7741	1.0747
1.0680	.2670	25.4297	27.7643	2.7764	1.0751
1.0690	.2672	25.4932	27.7877	2.7788	1.0754
1.0700	.2675	25.5569	27.8111	2.7811	1.0758
1.0710	.2677	25.6206	27.8345	2.7834	1.0762
1.0720	.2680	25.6844	27.8579	2.7858	1.0765
1.0730	.2682	25.7484	27.8813	2.7881	1.0769
1.0740	.2685	25.8124	27.9047	2.7905	1.0772
1.0750	.2687	25.8766	27.9280	2.7928	1.0776
1.0760	.2690	25.9408	27.9514	2.7951	1.0780
1.0770	.2692	26.0051	27.9748	2.7975	1.0783
1.0780	.2695	26.0696	27.9982	2.7998	1.0787
1.0790	.2697	26.1341	28.0216	2.8022	1.0790

Table 3: Computer Program of Computing Water Surface Profile

```
             PROGRAM DVODE(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)
000003       DIMENSION Y(90)
      C      THIS IS A FORTRAN IV PROGRAM TO COMPUTE THE WATER
      C      SURFACE PROFILE FOR SPATIALLY VARIED FLOW WITH
      C      INCREASING DISCHARGE OF RECTANGULAR, TRAPEZOIDAL, AND
      C      TRIANGULAR HIGHWAY GUTTERS
      C      ***********************************************
      C      *THE FOLLOWING VARIABLE NAMES ARE USED IN THIS PROBLEM*
      C      ***********************************************
      C      EL= TOTAL LENGTH OF GUTTER UNDER CONSIDERATION, IN FEET
      C      B= BREADTH OF CHANNEL BOTTOM, IN FEET
      C      Z= SIDE SLOPE OF CHANNEL, IN FEET PER FOOT
      C      SO= LONGITUDINAL GUTTER SLOPE, IN FEET PER FOOT
      C      EN= MANNINGS ROUGHNESS COEFFICIENT,N, FOR GUTTER
      C      ALFA= ENERGY COEFFICIENT
      C      DX= LENGTH OF INTEGRATION STEP, IN FEET
      C      PLSM= SIGN, DEPENDS ON DIRECTION IN WHICH COMPUTATIONS
      C            ARE DESIRED TO PROCEED.  THIS WILL HAVE A VALUE
      C            OF +1 WHEN COMPUTIONS PROCEED DOWNSTREAM AND -1
      C            WHEN THEY PROCEED UPSTREAM
      C      Q= TOTAL DISCHARGE, IN CFS
      C      Y(1)= INITIAL DEPTH OF WATER, IN FEET
      C      YDOTI= (DD/DX)I
      C      YDOTJ= (DD/DX)I+1
      C      DQDX= THE VARYING DISCHARGE ALONG THE GUTTER IN CFS PER
      C            FOOT
      C      Y(I)= DEPTH OF WATER AT ITH POINT ALONG THE GUTTER.  I
      C            IS THE SUBSCRIPT DENOTING DISTANCE DX*I
      C            DOWNSTREAM OR UPSTREAM FROM THE STARTING POINT
      C
      C
```

Table 3: Computer Program of Computing Water Surface Profile (continued)

```
000003            DO 400 L=1,2
000005            READ (1,101) EL,B,Z,SO,EN,ALFA,DX,PLSM,Q,Y(1),DQDX
000036            WRITE (2,103)
000042            I=0
000043            YY=1.079
000045            WRITE(2,100) PLSM,I,YY,Q
000060            N=EL/DX
000063            DO 300 I=1,N
000065            Q=Q+DQDX*DX*PLSM
000071            CALL YDOTF (B,SO,EN,Z,ALFA,Q,Y(I),SLOPE,DQDX)
000103            YDOTI=SLOPE*PLSM
000105            YDOTJ=YDOTI
000107            DO 200 J=1,15
000110            Y(I+1)=Y(I)+(YDOTI+YDOTJ)*DX*0.5
000116            TEMP=YDOTJ
000120            CALL YDOTF (B,SO,EN,Z,ALFA,Q,Y(I+1),SLOPE,DQDX)
000132            YDOTJ=SLOPE*PLSM
000134            N1=I+1
000136            IF (ABS (TEMP-YDOTJ)-.1E-05) 201,201,200
000143      200   CONTINUE
000145      201   WRITE(2,100) PLSM,I,Y(N1),Q
000161      300   CONTINUE
000164      400   CONTINUE
            C     ******************************************
            C     *THE FOLLOWING DATA ARE USED FOR THIS PROBLEM*
            C     ******************************************
```

Table 3: (Contd.)

```
        C      SET ONE
        C      EL=22.0 , B=0.0 , Z=0.5774 , SO=0.05 , EN=0.016
        C      ALFA=1.0 , DX=1.0 , PLSM=1.0 , Q=2.80 , Y(1)=1.079
        C      DQDX=0.1
        C      SET TWO
        C      EL=28.0 , B=0.0 , Z=0.5774 , SO=0.05 , EN=0.016
        C      ALFA=1.0 , DX=1.0 , PLSM=-1.0 , Q=2.80 , Y(1)=1.079
        C      DQDX=0.1
000166  100    FORMAT(  7X,F5.1,8X,I5,F17.3,F16.3)
000166  101    FORMAT(8F10.4/3F10.4)
000166  102    FORMAT (1H1,11F10.4)
000166  103    FORMAT (1H1,7(/),7X,4HPLSM,12X,1HI,12X,6HY(I+1),12X,1HQ,//)
000166         CALL EXIT
000167         END
```

Table 3: (Contd.)

```
              SUBROUTINE YDOTF (B,S,EN,Z,AA,Q,YY,SLOPE,DQDX)
000014        DIMENSION YY(20)
000014        ATERM=(EN*Q)**2*(B+2.0*YY*SQRT(1.+Z*Z))**(4./3.)
000035        BTERM=2.21*((B+Z*YY)*YY)**(10./3.)
000046        CTERM=(2.*Q*DQDX )/(32.17*((B+Z*YY)*YY)**2)
000056        DTERM=AA*Q*Q*(B+Z*YY)
000062        ETERM=32.17*((B+Z*YY)*YY)**3
000066        FTERM= (S-(ATERM/BTERM)-CTERM)/(1.-DTERM/ETERM)
000075        SLOPE=FTERM
000077        RETURN
000100        END
```

Table 4: Final Result of Water Surface Elevation Calculation

PLSM	I	Y(I+1)	Q
-1.0	0	1.079	2.800
-1.0	1	1.065	2.700
-1.0	2	1.051	2.600
-1.0	3	1.039	2.500
-1.0	4	1.027	2.400
-1.0	5	1.015	2.300
-1.0	6	1.004	2.200
-1.0	7	.992	2.100
-1.0	8	.981	2.000
-1.0	9	.969	1.900
-1.0	10	.957	1.800
-1.0	11	.945	1.700
-1.0	12	.933	1.600
-1.0	13	.920	1.500
-1.0	14	.906	1.400
-1.0	15	.892	1.300
-1.0	16	.877	1.200
-1.0	17	.862	1.100
-1.0	18	.846	1.000
-1.0	19	.829	.900
-1.0	20	.810	.800
-1.0	21	.791	.700
-1.0	22	.770	.600
-1.0	23	.747	.500
-1.0	24	.722	.400
-1.0	25	.693	.300
-1.0	26	.660	.200
-1.0	27	.621	.100
-1.0	28	.571	-.000

Table 4: (Contd.)

PLSM	I	Y(I+1)	Q
1.0	0	1.079	2.800
1.0	1	1.092	2.900
1.0	2	1.107	3.000
1.0	3	1.123	3.100
1.0	4	1.142	3.200
1.0	5	1.163	3.300
1.0	6	1.188	3.400
1.0	7	1.214	3.500
1.0	8	1.244	3.600
1.0	9	1.276	3.700
1.0	10	1.310	3.800
1.0	11	1.346	3.900
1.0	12	1.384	4.000
1.0	13	1.423	4.100
1.0	14	1.463	4.200
1.0	15	1.505	4.300
1.0	16	1.547	4.400
1.0	17	1.590	4.500
1.0	18	1.634	4.600
1.0	19	1.679	4.700
1.0	20	1.723	4.800
1.0	21	1.769	4.900
1.0	22	1.815	5.000

Length of Inlet, x (ft)	Width of Inlet, $y_o \tan \theta_o$ (ft)
0.00	4.70
0.09	4.00
0.34	3.50
0.43	3.00
0.70	2.50
0.98	2.00
1.31	1.50
1.69	1.00
2.08	0.50
2.54	0.00

Table 5: Length of Inlet, x, and Corresponding Width of the Inlet for Removal of all Water Flowing in the Gutter

NOMENCLATURE

A	cross sectional area of the gutter, sq ft
C	factor of flow resistance in Chezy's formula
C_d	discharge coefficient of the inlet
C_e	weir discharge coefficient
d	elevation between water surface and datum, ft
D_e	height of the side inlet above the gutter, ft
E	specific energy in the gutter, ft
F	hydrostatic force, lb
f	Darcy-Weisbach friction factor
F_f	friction force, lb
g	acceleration of gravity, fpsps
H	metering differential, ft of water
J	ratio of the open area to the total area of an inlet
k	conveyance of the gutter
K	characteristic coefficient of an inlet
L_o	theoretical length of the inlet required to catch the entire flow, ft
M	mass of flowing water within the control volume, lbm
n	resistance coefficient, customarily termed Manning's n
P	wetted-perimeter, ft
q	increment of rainfall inflow per unit length, cfs per foot
Q	discharge, cfs
R	hydraulic radius, ft
S	slope of the energy line
S_f	friction slope

S_o	bed slope of the gutter
T	top width of the flow, ft
v	local velocity, fps
V	mean velocity, fps
V_o	average velocity of flow corresponding to L_o, fps
W	weight of water in the control volume, lb
y_m	hydraulic depth in the gutter, ft
y_o	depth of flow in the gutter corresponding to L_o, ft
z	elevation from datum to gutter vottom, ft
-dQ/dx	discharge withdrawn through a length of dx of the inlet
dy/dx	water surface profile
α	momentum correction coefficient, $\int v^2 dA/V^2 A$
β	energy correction coefficient, $\int v^3 dA/V^3 A$
γ	specific weight of water, pcf
θ	angle between the bottom of the gutter and the vertical

REFERENCES

1. Beij, K. H.
   FLOW IN ROOF GUTTERS, Journal of Research, US NBS, Vol. 12, pp. 193-213, (February 1934).

2. Camp, T. R.
   LATERAL SPILLWAY CHANNELS, Trans., ASCE, Vol. 105, pp. 606-617, (1955).

3. Chow, V. T.
   OPEN-CHANNEL HYDRAULICS, McGraw-Hill Book Co., Inc., New York, pp. 98-114, 327-349, (1959).

4. Chow, V. T.
   A NOTE ON THE MANNING FORMULA, Trans., AGU, Vol. 36, No. 4, pp. 688-705, (August 1955).

5. Chow, V. T.
   INTERGRATING THE EQUATION OF GRADUALLY VARIED FLOW, Proc., ASCE, Vol. 81, pp. 830-862, (November 1955).

6. Daugherty, R. L. and Ingersoll, A. C.
   FLUID MECHANICS WITH ENGINEERING APPLICATIONS, 5th Edition, McGraw-Hill Book Co., Inc., New York, pp. 236-239, (1954).

7. Doland, J. J. and Chow, V. T.
   DISCUSSION ON RIVER CHANNEL ROUGHNESS, by H. A. Einstein and N. L. Barbarossa, Trans., ASCE, Vol. 117, pp. 1134-1139, (1952).

8. Einstein, H. A.
   FORMULAS FOR THE TRANSPORT OF BED LOAD, Trans., ASCE, Vol. 107, pp. 561-597, (1942).

9. Escoffier, F. F.
   TRANSITIONAL PROFILES IN NON-UNIFORM CHANNELS, Trans., ASCE, Vol. 123, pp. 43-65, (1958).

10. Frazer, W.
    THE BEHAVIOR OF SIDE WEIRS IN PRISMATIC RECTANGULAR CHANNELS, Proc., Inst. of Civil Engrs., London, Vol. 6, pp. 305-328, (February 1957).

11. Giorgio, N.
    OPERATION AND DESIGN OF BOTTOM INTAKE RACKS, Proc. of the 6th General Meeting, Int. Assoc. of Hydr. Res., The Hague, Vol. 3, pp. C17-1 to C17-11, (1955).

12. Guillou, J. C.
THE USE AND EFFICIENCY OF SOME GUTTER INLET GRATES, Univ. of Illinois, Engrg. Exp. Sta., Bull. No. 450, (July 1959).

13. Henderson, F. M.
OPEN-CHANNEL FLOW, Macmillan Co., New York, pp. 96-99, 1966. (1966).

14. Hinds, J.
SIDE CHANNEL SPILLWAYS: HYDRAULIC THEORY, ECONOMIC FACTOR, AND EXPERIMENTAL DETERMINATION OF LOSSES, Trans., ASCE, Vol. 89, pp. 881-927, (1926).

15. Jansen, R. B.
SURFACE CURVES FOR STEADY NON-UNIFORM FLOW, Trans., ASCE, Vol. 117, pp. 1091-1120, (1952).

16. Johns Hopkins University
THE DESIGN OF STORM-WATER INLETS, Dept. of San. Engrg. and Water Res., Rept. of the Storm Drainage Res. Com., Baltimore, Maryland, (June 1956).

17. Jones, L. E., Diskin, M. H., and Chow, V. T.
DISCUSSION ON SOLVING THE EQUATION OF UNIFORM FLOW, by W. F. Pickard, Proc., ASCE, Vol. 90, No. HY1, pp. 319-327, (January 1964).

18. Keulegan, G. H.
SPTATIALLY VARIABLE DISCHARGE OVER A SLOPING PLANE, Trans., AGU, pp. 956-159, (1944).

19. Larson, C. L. and Straub, L. G.
GRATE INLETS FOR SURFACE DRAINAGE OF STREETS AND HIGHWAYS, Univ. of Minnesota, St. Anthony Falls Hydr. Lab., Bull. No. 2, (June 1949).

20. Li, W. H.
HYDRAULIC THEORY FOR DESIGN OF STORM-WATER INLETS, Highway Res. Board, Vol. 33, pp. 83-91, (1954).

21. Li, W. H.
OPEN-CHANNELS WITH NON-UNIFORM DISCHARGE, Trans., ASCE, Vol. 120, pp. 155-180, (1955).

22. Morris, H. M.
APPLIED HYDRAULICS IN ENGINEERING, The Ronald Press Co., New York, pp. 86-88, (1963).

23. Pickard, W. F.
SOLVING THE EQUATIONS OF UNIFORM FLOW, Proc., ASCE, Vol. 89, No. HY4, pp. 23-37, (June 1963).

24. Prasad, R.
NUMERICAL METHOD OF COMPUTING FLOW PROFILES, Proc., ASCE, Vol. 96, No. HY1, pp. 75-86, (January 1970).

25. Selby, S. M.
STANDARD MATHEMATICAL TABLES, 16th Edition, The Chem. Rubber Co., pp. 403-405, (1968).

26. Smith, K. V. H.
CONTROL POINT IN A LATERAL SPILLWAY CHANNEL, Proc., ASCE, Vol. 93, No. HY3, pp. 17-34, (May 1967).

27. Streeter, V. L.
FLUID MECHANICS, 4th Edition, McGraw-Hill Book Co., Inc., New York, pp. 251-253, (1966).

28. Collinge, V. K.
THE DISCHARGE CAPACITY OF SIDE WEIRS, Proc., Inst. of Civil Engrg., London, Vol. 6, pp. 288-304, (February 1957).

29. Wasley, R. J.
HYDRODYNAMICS OF FLOW FROM ROAD SURFACES INTO CURB INLETS, Stanford Univ., Dept. of Civil Engrg., Rept. No. 6, (November 1960).

30. Williams, G. P.
MANNING FORMULA-A MISNOMER, Proc., ASCE, Vol. 96, No. HY1, pp. 193-200, (January 1970).

31. William, J. B. and Woo, D. C.
HYDRAULIC DESIGN OF DEPRESSED CURB-OPENING INLETS, Hydr. Design Series 6, U. S. Bureau of Public Roads, Part III, Depressed Curb-Opening Inlets, pp. 61-79, (1964).

32. Yen, B. C. and Wenzel, H. G., Jr.
DYNAMIC EQUATIONS FOR STEADY SPATIALLY VARIED FLOW, Proc., ASCE, Vol. 96, No. HY3, pp. 801-813, (March 1970).

VITA

The author was born in Sze-Chwan, China on November 15, 1944. He is the son of Mr. and Mrs. Lee Chuh. He received his primary and secondary education in Pin-Tung, Taiwan, China.

Upon graduation from high school the author studied Civil Engineering at Chung-Yuan College of Science and Engineering, and received his Bachelor of Engineering degree in Civil Engineering in June 1967.

After one year of ROTC Training as a Chinese Army Engineer he was awarded a Research Assistantship to Lehigh University, from which he graduated in 1970 receiving the degree of Master of Science in Civil Engineering.